The influence of culture on mathematics teaching and learning

Insights from FoNS and various other projects

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- In this talk, I want to talk about the ways in which culture influences all aspects of mathematics teaching and learning
- I want to do it in ways that are light-hearted but informative
- I want to talk a little about PISA
- And, in so doing, expose its repeated failings
- But first, I want to start with a little story, a classroom anecdote

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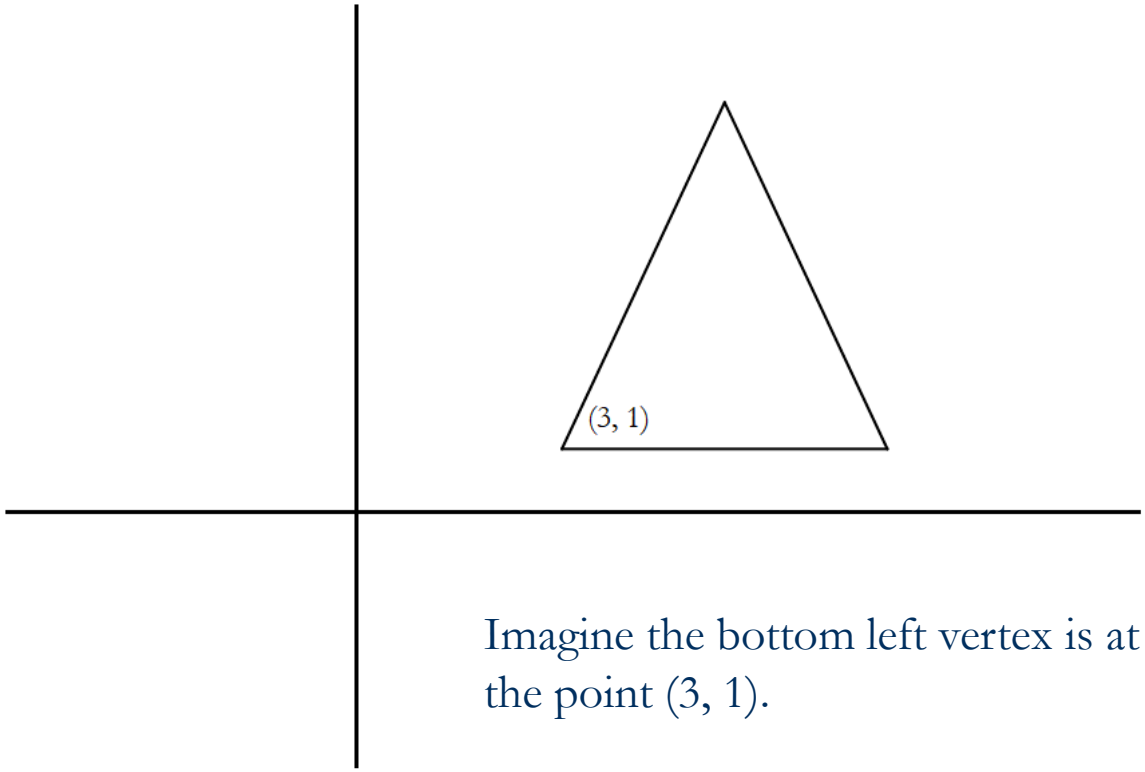
- Some years ago, I was observing a grade 7 lesson in Hungary
- The teacher drew a triangle…
- And then added some conditions
  - It is an isosceles triangle with an area of 9 units squared
  - All the vertices are on integer grid points
  - One of the vertices is at the point (3,1)
  - How many such triangles are there?
- After about fifteen minutes of working in pairs, an answer was agreed and a student came to the board.

Suppose such a triangle exists and



Imagine the bottom left vertex is at the point (3, 1).

We can rotate the triangle about this point through 90 degrees

Because it will still lie on grid points

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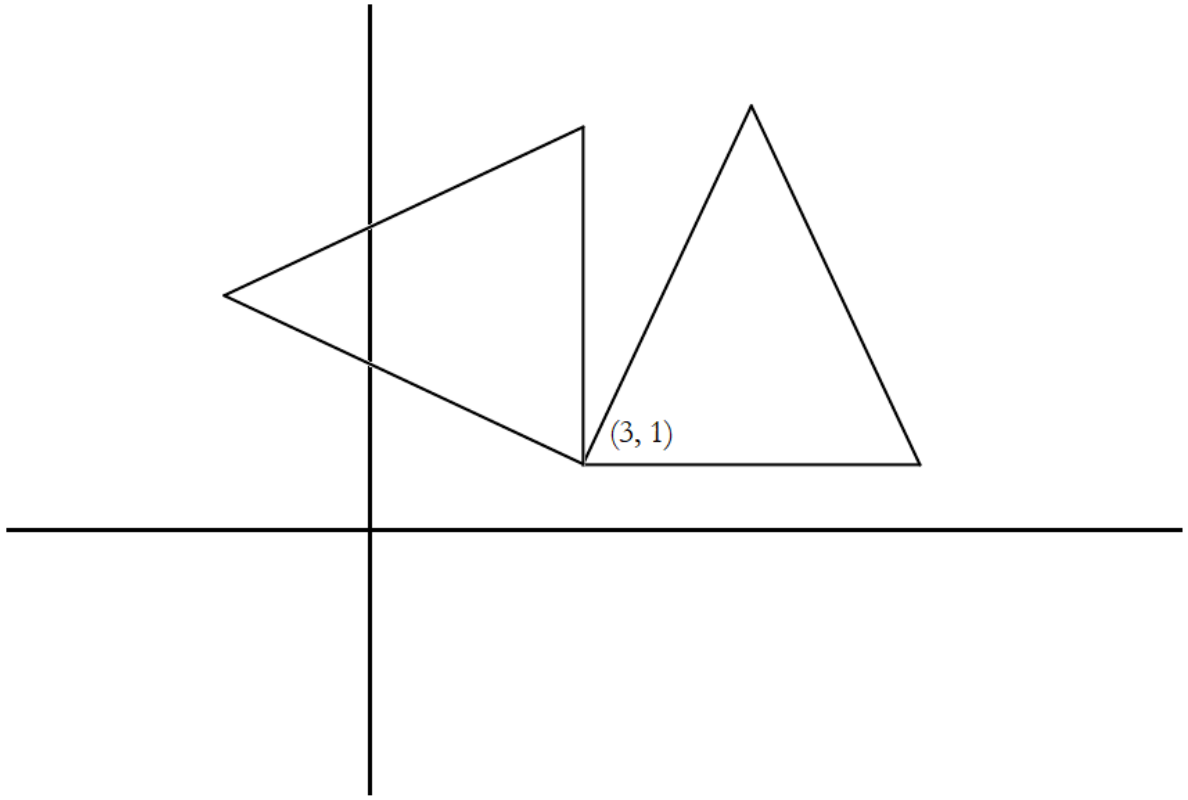

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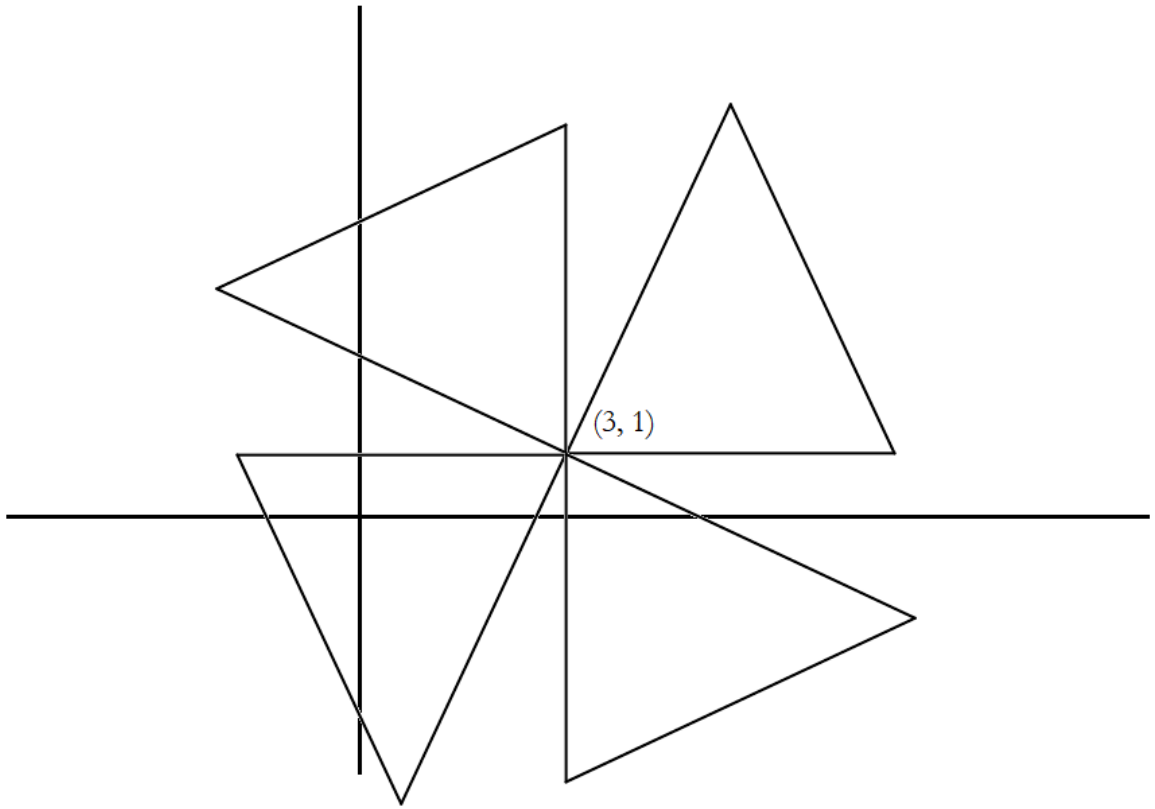

This will give us four possible triangles

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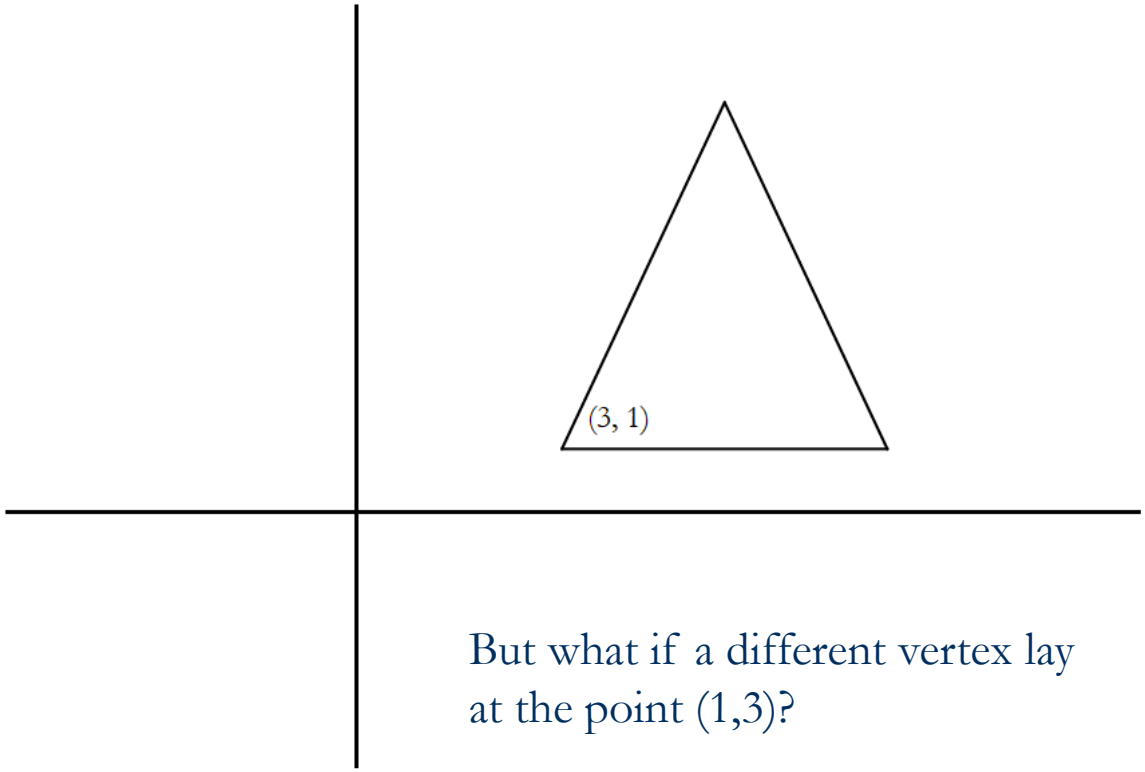

But what if a different vertex lay at the point (1,3)?

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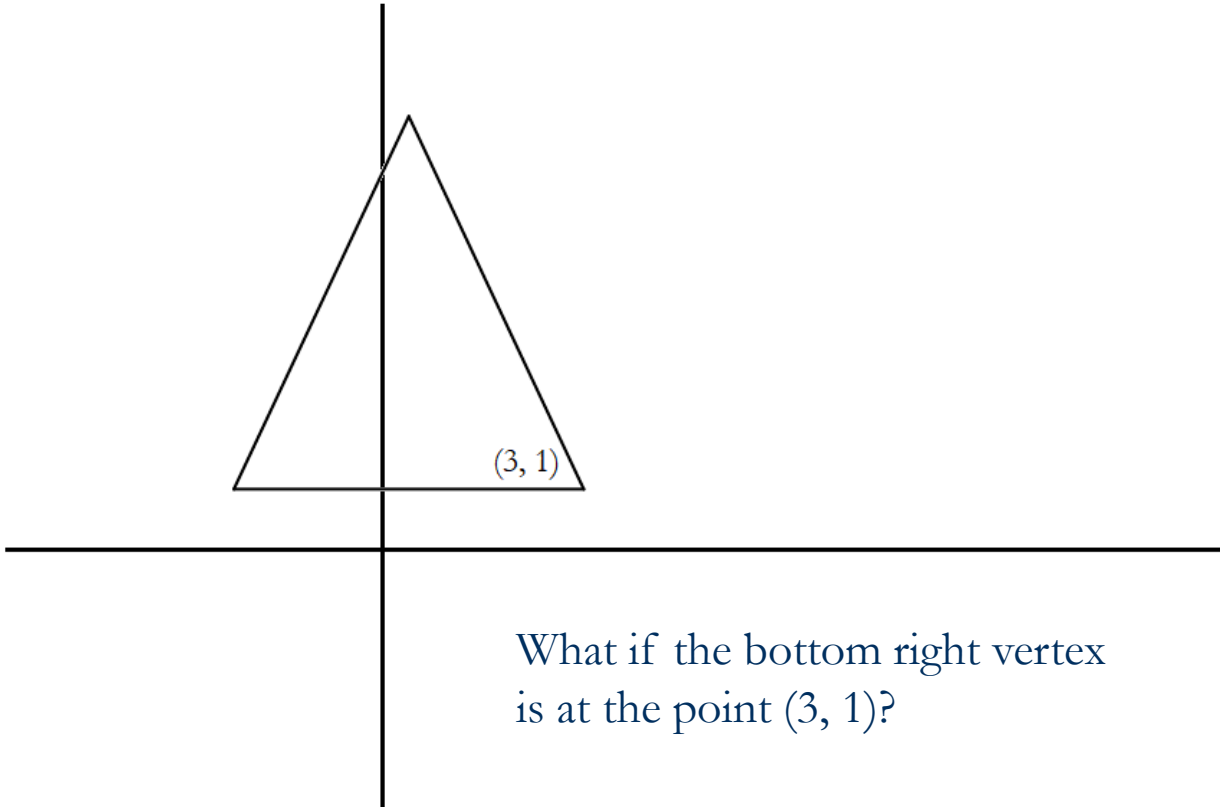

What if the bottom right vertex is at the point (3, 1)?

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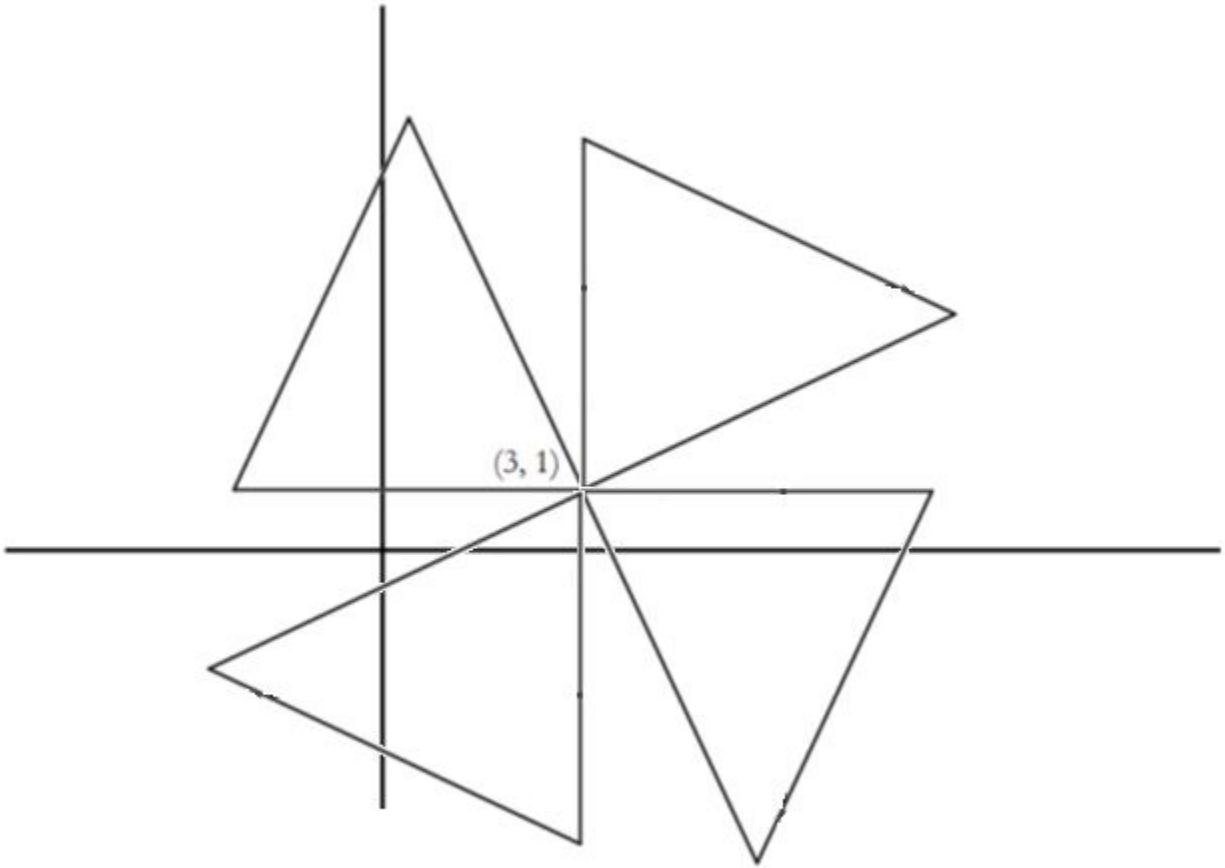

So, this gives another four possibilities

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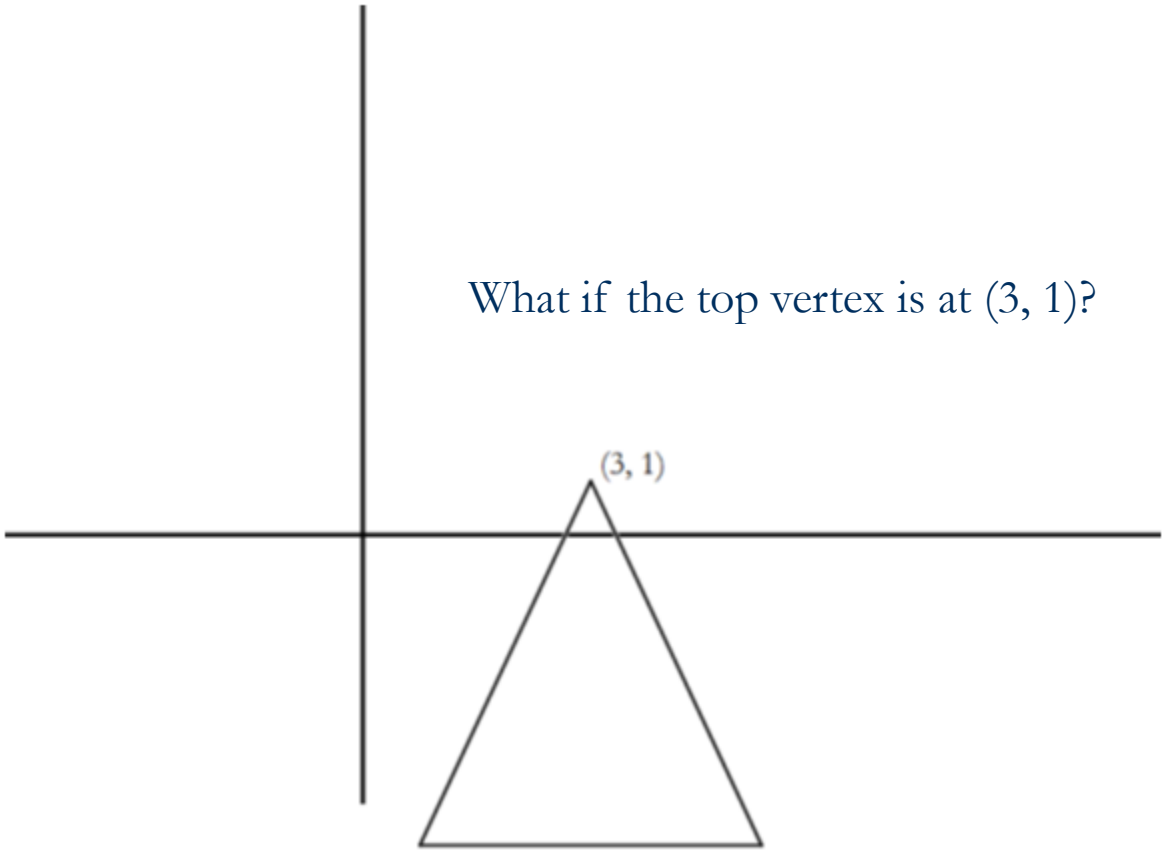
What if the top vertex is at (3, 1)?



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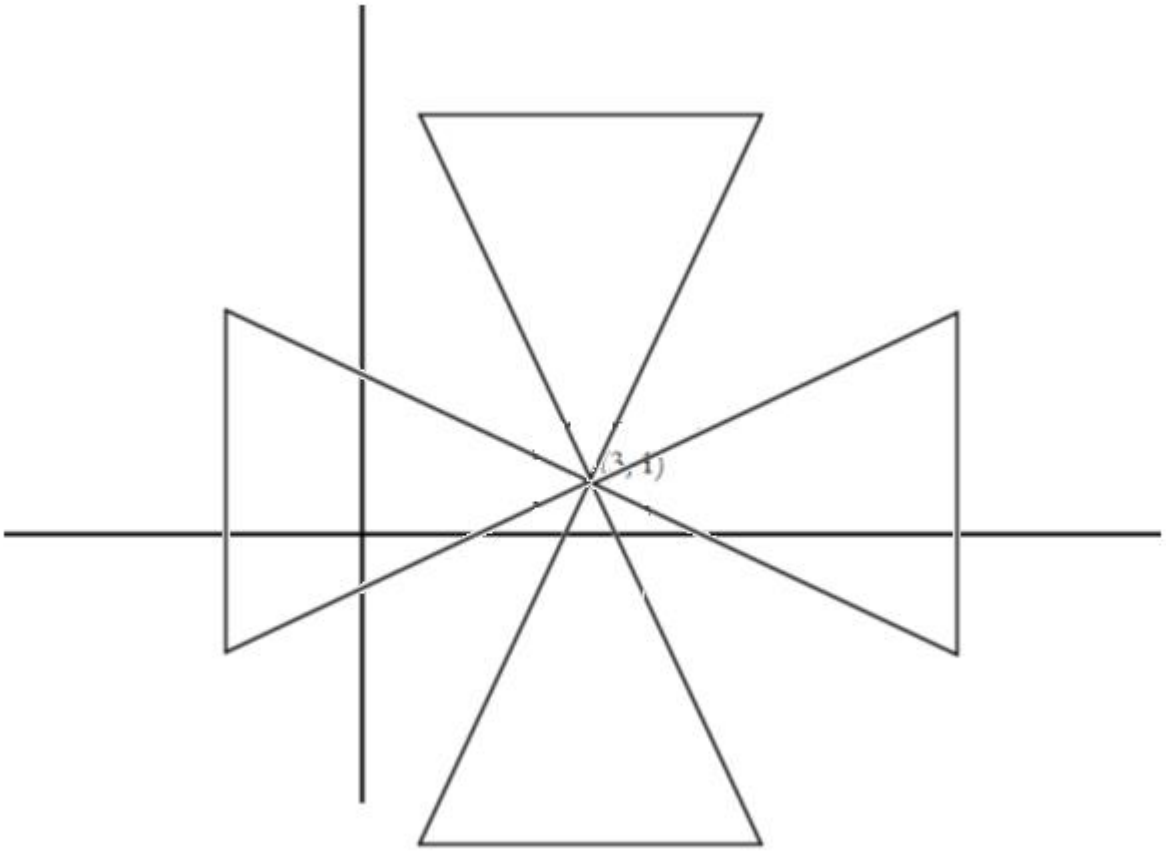

This gives another four possibilities

So, for any triangle that exists, there are 12 possible triangles

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- Now, does such a triangle exist?
- If the area is 9 then the base multiplied by the height must be 18.
- And because all three vertices are on grid points, both the base and the height must be integers. So, we have these possibilities.

So, why am I telling this story?

- But, because the triangle is isosceles, the base must also be even.
- Only three of these triangles have an even base. So, 3 times 12 equals 36
- And this leads me to an earlier European study…

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Teachers' observed learning goals in whole class situations

	Flanders	England	Hungary	Spain
Conceptual knowledge	71	79	64	77
Connected knowledge	17	1	40	15
Procedural knowledge	57	53	51	68
Mathematical efficiency	13	10	36	15
Problem-solving	7	20	31	39
Reasoning	35	30	45	25

The percentage of each country's episodes to which each code was applied

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What is the purpose of school mathematics?

Mogens Niss wrote that,

> it is very difficult indeed to identify and locate the real goals of mathematics education in any given society. Firstly, it is often the case that these goals are not made explicit. What we can observe is the presence and the reality of mathematics education in its various formats and shapes, whereas the goals, like the underlying reasons and driving forces, are not directly observable (Niss, 1996, p. 17).

So, is the purpose of school mathematics to

- prepare children for work?
- prepare children for later study?
- support the teaching of other subjects?
- create enjoyment?
- help students become better citizens?
- help people manage their money efficiently?

In Spain, it is to help students solve problems related to daily life and the workplace (García, Maass & Wake, 2013)

In Japan it has traditionally been unrelated to any sense of the real world (Yanagimoto & Yoshimura, 2001).

Such matters lead us neatly to the OECD and PISA.

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PISA: An unwanted and unnecessary intervention

- PISA assesses age 15 children on literacy, mathematics and science.
- It (cl)aims to go beyond classroom mathematics to assess those situations people face when going about their daily lives (OECD, 2003).
- The OECD aims "to promote policies that will improve the economic and social well-being of people around the world".

Mathematics	2000	2003	2006	2009	2012	2015	2018
Denmark	514	514	513	503	500	511	509
Finland	536	544	546	541			

- During this time, 15 000 German-speakers visited Finland and…
- Swedes started importing Finnish mathematics textbooks

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Some things change

Mathematics	2000	2003	2006	2009	2012	2015	2018
Denmark	514	514	513	503	500	511	509
Finland	536	544	546	541	519	511	507

- With the bursting of the Finnish bubble, a new player emerged

Singapore	***	***	***	562	573	564	569

- Can we trust PISA? Do we want to be like Singapore?

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Singapore: Some interesting facts

- Singapore is effectively a one party state, sustained by good public housing (Ortmann & Thompson, 2018), security and prosperity (Wong & Huang, 2010).
- However, in order to benefit from such things, people seem prepared to accept
  - poor human and political rights (Han, 2017; Nasir & Turner, 2013),
  - an uncritical media (Tushnet, 2015),
  - corporal punishment as a means of deflecting dissent (Druzin & Wan, 2015)
- And at school
  - corporal punishment of boys (Gershoff, 2017),
  - academic stress prompted by high-stakes testing (Ang et al., 2009),
  - much supplementary private tuition (Aun et al., 2006).
- But worse…

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Singapore: Some more interesting facts

- In 2018, the permanent resident population was 3,994,300
- Of these, 2,628,800 people were in the age range 20-64 and represented the likely resident working population.
- In addition, 1,386,000 people were foreign employees who earned too little to be permanent residents or have family members join them.
- In short, **at least** one third of all people who work in Singapore are not permitted to have their children live with them.
- What if…?

Mathematics	2000	2003	2006	2009	2012	2015	2018
Denmark	514	514	513	503	500	511	509

- Can we trust PISA?

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PISA's cynical manipulation of the public

- When discussing strategies for raising German politicians' awareness of PISA's results, Andreas Schleicher, PISA's head, commented that

> going to the people in charge isn't going to change the system. And I actually changed strategy and thought, I'm going to go to work with the media… and that has created a public demand for better education… parents knocked on the door of schools, schools knocked on the door of local administrators – and a week after this the Chancellor in Germany went public about this, saying what they needed to do…. (Schleicher, 2015 cited in Grey & Morris, 2019, p. 111).

- In this way, the OECD deliberately created a discourse of 'scandalisation' to justify systemic change (Baird et al., 2016).
- And this certainly worked in Sweden

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Scandalised Sweden

- DN: *Sweden is the worst in the class* (Sverige sämst i klassen)
- Aftonbladet: *The Swedish school is sinking* (Svensk skola sjunker).
- Even the Royal Swedish Academy of Engineering Sciences fell into the trap, producing a report titled:

  Educational performance in Swedish schools is plummeting – what are the facts?
- After PISA 2012, the *scandalised* authorities invited the OECD to undertake a review of Swedish education. It concluded that
  - Sweden should implement a comprehensive education reform to bring about system-wide change and strengthen the performance of all Swedish schools and students (OECD, 2015, p. 8).
- But there is a mass of evidence, ignored by everyone, showing Sweden to be a very successful system.
- Can we trust PISA?

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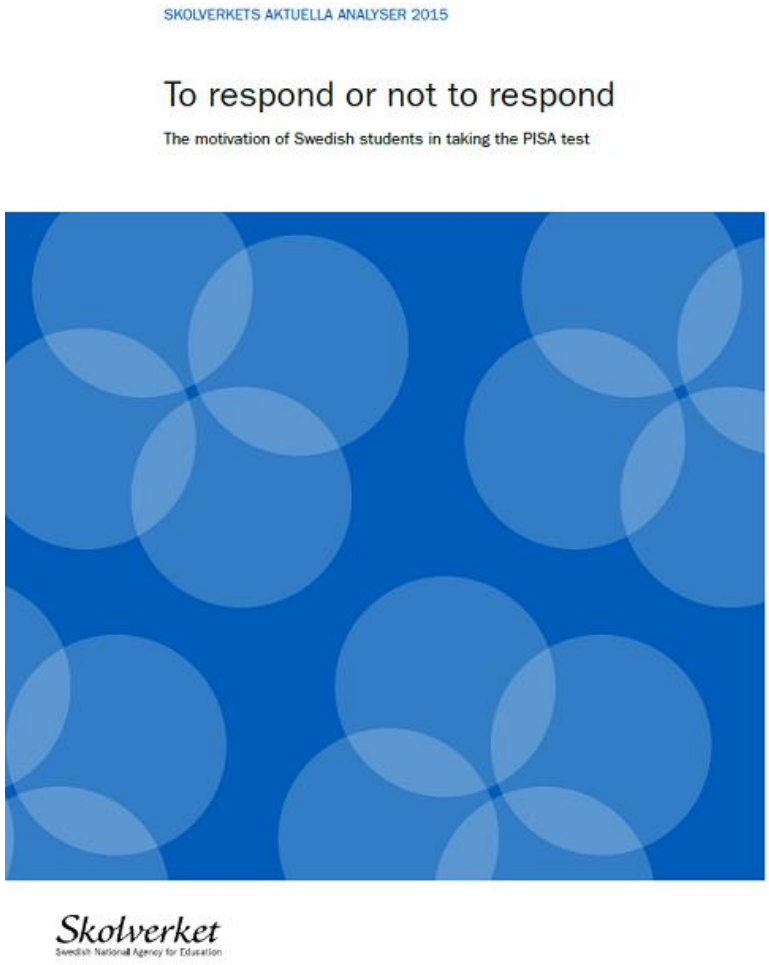
For example



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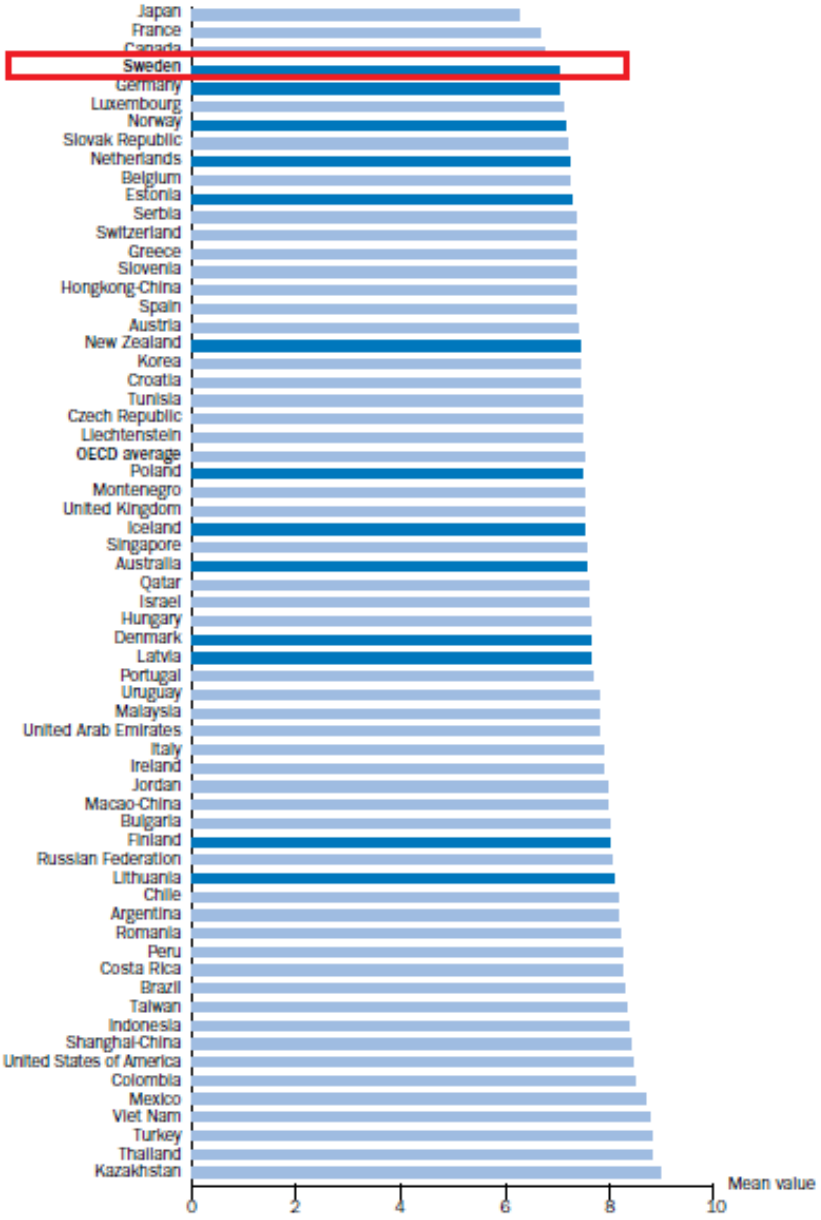
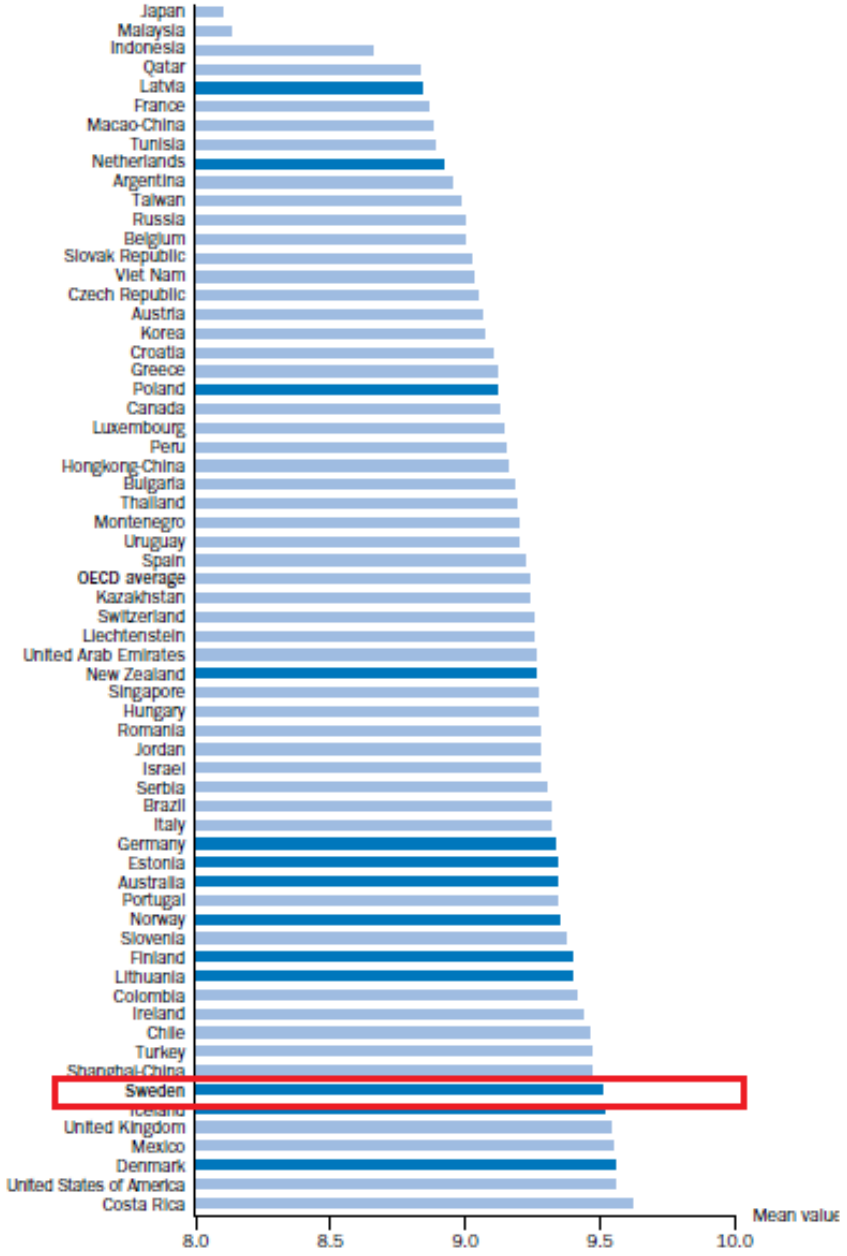
Mean value of reported effort in PISA 2012.



Mean value of estimated effort if PISA was going to count towards a grade



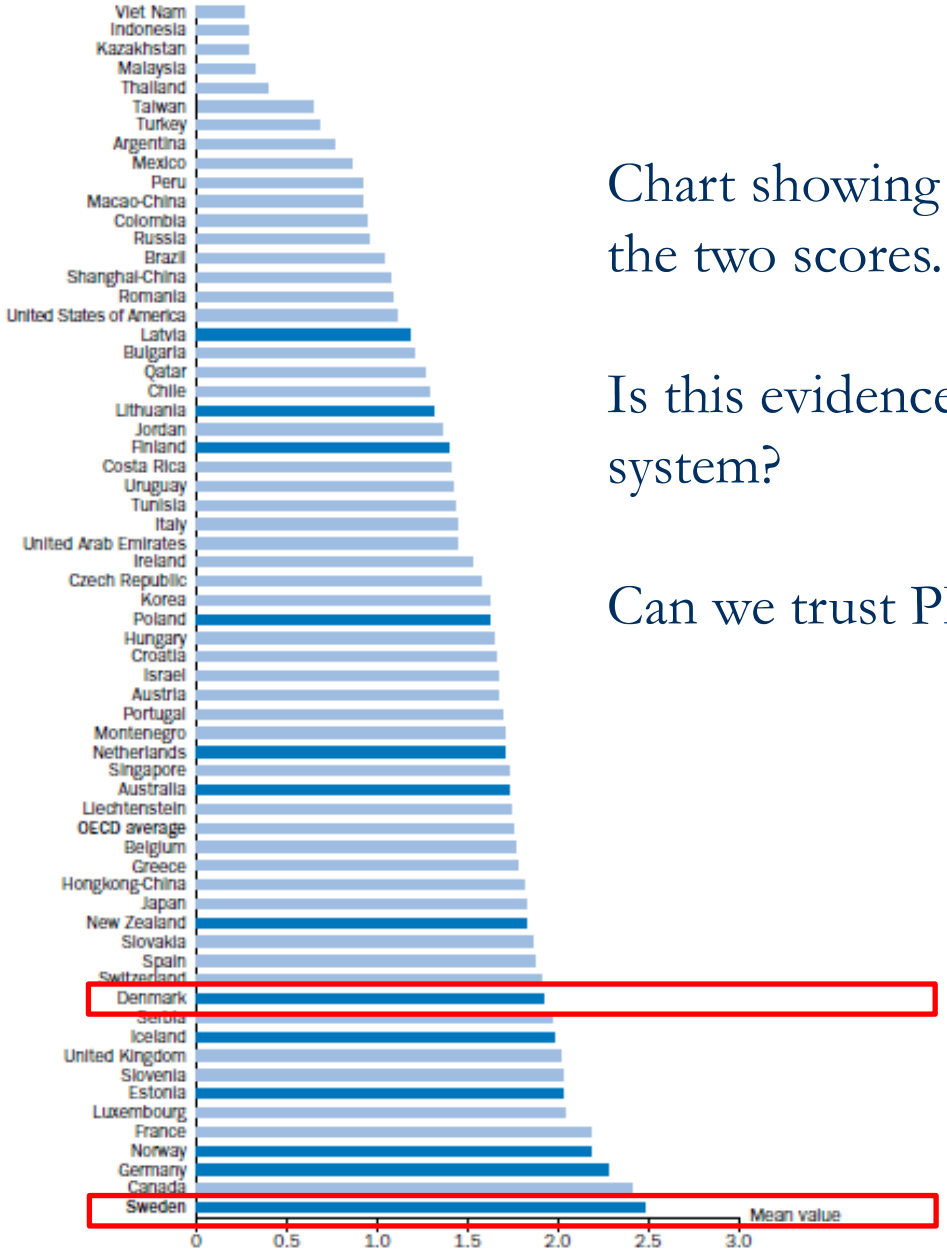


Chart showing the difference between the two scores.

Is this evidence of a failing educational system?

Can we trust PISA?

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World Economic Forum (≈150 nations)

The Global Competitiveness Index

Sweden	2008	2010	2012	2014	2016	2018
GCI	4	2	4	10	6	9
Innovation	6	3	5	7	5	5

Denmark	2008	2010	2012	2014	2016	2018
GCI	3	9	12	13	12	10
Innovation	7	9	12	9	10	12

And, don't forget, Denmark and Sweden have genuine comprehensive education systems, with very few elite private schools.

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World Economic Forum (≈150 nations)

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Denmark	2008	2010	2012	2014	2016	2018
GCI	3	9	12	13	12	10
Innovation	7	9	12	9	10	12

The 2019 annual English language proficiency index.

They conclude that countries with high English proficiency are fairer and more open

01	Netherlands	70.27
02	Sweden	68.74
03	Norway	67.93
04	Denmark	67.87

Are these failing education systems?

Can we trust PISA?

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VOLVO
ALFA LAVAL
Electrolux
H&M
FJÄLL RÄVEN
SECURITAS
SKANSKA
Can we trust PISA?
ABB
Arla
ERICSSON
SCANIA
PROTECTS WHAT'S GOOD
Tetra Pak
ABSOLUT
Country of Sweden
VODKA
THULE
SWEDEN
SKF
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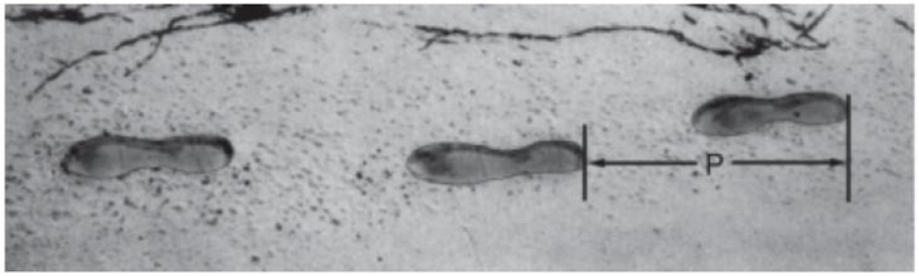

The picture shows the footprints of a man walking. The pace-length *P* is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P

where, n = number of steps per minute, and P = pace-length in metres.

1. If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pace-length? Show your work.
2. Bernard knows his pace-length is 0.80 metres. The formula applies to Bernard's walking. Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.

But this is a massively problematic task – let's just transpose the formula

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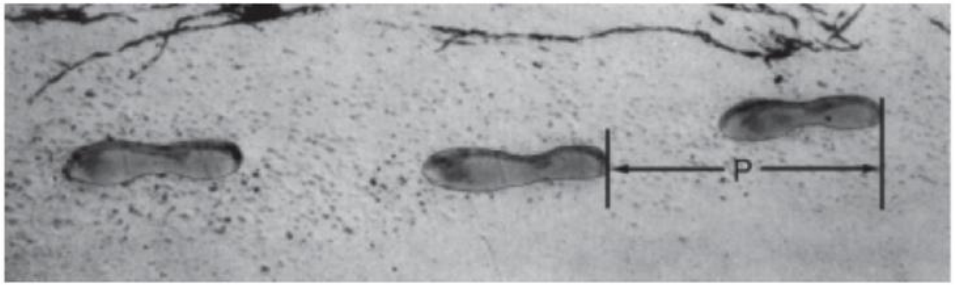

$$n = 140 \cdot P$$

$$35 = 140 \cdot 0{,}25$$

$$70 = 140 \cdot 0{,}50$$ (This is Heiko)

$$105 = 140 \cdot 0{,}75$$

$$140 = 140 \cdot 1{,}00$$

$$280 = 140 \cdot 2{,}00$$

Can we trust PISA?

And this takes me to the foundational number sense (FoNS) project

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The Foundational Number Sense (FoNS) project

Judy Sayers, formerly Stockholm University, now Leeds University

Jöran Petersson, formerly Stockholm University, now Malmö University

Eva Rosenqvist, Stockholm University

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The Foundational Number Sense (FoNS) project

- We analysed nearly 400 articles from research in mathematics education, psychology, special educational needs and generic education.
- Our aim was to design "a simple to operationalise framework… with the potential to inform teacher education, facilitate classroom evaluations and provide a warranted tool for use in cross-cultural studies"
- FoNS comprises eight, curriculum independent, categories of competence, each requiring instruction, that all year-one children should acquire if they are to become successful learners of mathematics.
- FoNS connects the innate competences of the approximate number system (Dehaene, 1997) with the number-related competences "required by all adults regardless of their occupation" (McIntosh et al., 1992, p.3).

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	Teachers encourage, in relation to the numbers 0-20, year-one children to…
Number recognition	identify, name and write number symbols
Systematic counting	count systematically, forwards and backwards, from arbitrary starting points
Number and quantity	engage with the one-to-one correspondence between number and quantity
Quantity discrimination	compare magnitudes and deploy language like ‘bigger than’ or ‘smaller than’
Different representations of number	recognise and make connections between different representations of number
Estimation	estimate the numerosity of a set of objects, the results of a calculation or the location of number on a number line
Simple arithmetical operations	undertake simple addition and subtraction operations
Number patterns	recognise and extend number patterns, identify missing numbers

The project's initial goals

- Interviews with year one teachers in England and Sweden
  - Twenty teachers in each country
  - Semi-structured interview focused on a range of topics
  - Multiple outcomes and more on the way
- Develop a questionnaire for use with teachers internationally
- Interviews with year one parents in England and Sweden
  - Sweden: three schools contacted and twenty parents interviewed
  - England: 200 schools contacted, advertisements in local newspapers, appeals on a popular parents' website, posters in supermarkets and zero interviews
- Develop a questionnaire for use with parents internationally
- Observe year one teachers in England and Sweden
  - But then… COVID
- Analyses of year one textbooks for FoNS-related opportunities

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Teachers' perspectives on parental involvement 1

- Implicit parental involvement
  - All teachers argued that parents should present positive attitudes towards mathematics, even if their own experiences as learners were negative

- Explicit parental involvement
  - English teachers expect parents to provide various informal learning opportunities to complement schools' efforts.
  - Swedish teachers see the role of the home as less about providing specific activities than ensuring the supportive environment that would encourage the acquisition of positive attitudes.
  - In other words, English teachers see the home as an extension of school, while their Swedish colleagues see it as separate from school.

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Teachers' perspectives on parental involvement 2

- Explicit parental involvement: communication
  - English teachers' communications tell parents focused on what parents should do to help children meet their targets.
  - Swedish teachers' communication update parents on what children are doing
  - In sum, English teachers' communication was instructive, while for Swedish teachers it was informative.
- Explicit parental involvement: homework
  - All English teachers advocated homework as essential if children are to meet their targets
  - Some Swedish teachers set homework and some did not, but all see homework as a potential threat to educational equity.
  - In sum, English teachers expect parents to be active in supporting their children's homework completion, while Swedish teachers expect them to be passive.

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Swedish parents perspective on homework

- All parents reported the existence and their approval of reading homework, seeing helping their children learn to read as a natural responsibility
- A fifth of parents opposed mathematics homework on the basis of equity
- A third of parents wanted homework but were concerned about the potential challenge to equity
- Half of all parents wanted homework
  - Because it helped them keep up with what children are doing
  - It establishes routines for later learning
  - It helps with routine practice
  - Although half of this group want homework in a form the prevented any need for explicit parental help.

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Swedish parent-initiated mathematics activities

- Most parents argued that playing board games not only enhances children's learning of mathematics but creates time for shared enjoyment
- Most parents spoke of informal contextualised activities located in the everyday world – conversations around cooking, counting when out walking, identifying numbers in different contexts
- Such activities, we argue, complement rather than conflict with what schools are trying to achieve. However, problematically
- One third of parents spoke about how they encourage their children to learn multiplication tables.
- In sum, parents' and teachers' views on both homework and home-based activities seem in unnecessary conflict.

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The results of scandalisation: Imported textbooks

- The curriculum independent nature of FoNS makes it an ideal tool for analysing textbooks.
- We recently compared the distribution of tasks focused on the eight FoNS competences in three year-one textbooks currently used in England
  - Abacus (English-authored)
  - Maths: No Problem (Singaporean-authored)
  - Inspire Maths (Singaporean-authored)
- The two Singaporean textbooks have been promoted by the English education authorities
- Both will have passed the Singaporean Ministry's evaluation criteria for use in the country.
- So, what do such analyses tell us?

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Comparing three English textbooks

	Abacus (1524)		Maths No Problem (1955)		Inspire Maths (2494)	
	n	%	n	%	n	%
Number recognition	737	48	685	35	740	30
Systematic counting	250	16	269	14	346	14
Number and quantity	338	22	371	19	356	14
Quantity discrimination	57	4	120	6	124	5
Different representations	274	18	370	19	341	14
Estimation	6	0	0	0	0	0
Simple arithmetical operations	573	38	423	22	464	19
Number patterns	80	5	0	0	14	1

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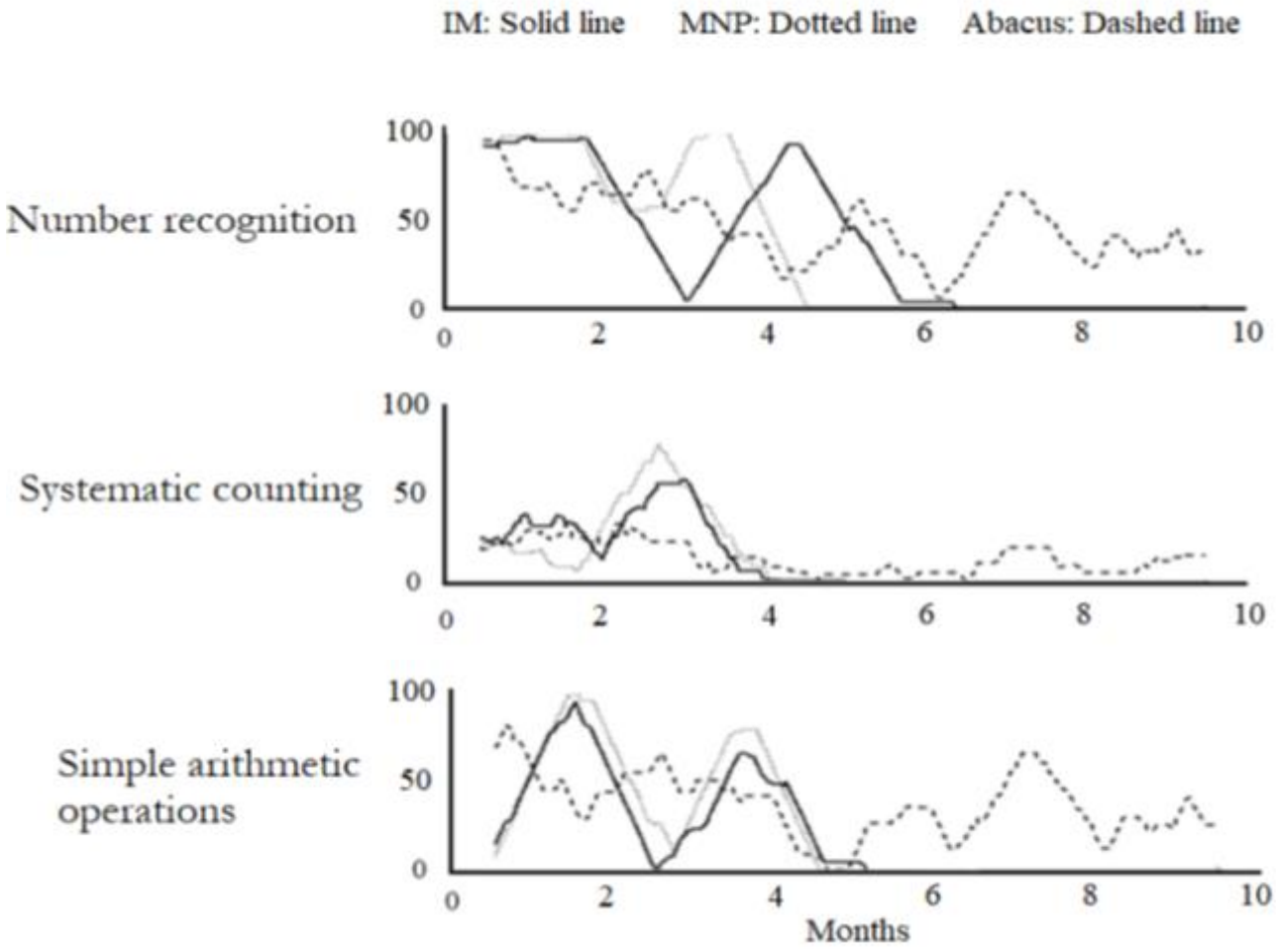
How do such things play out?



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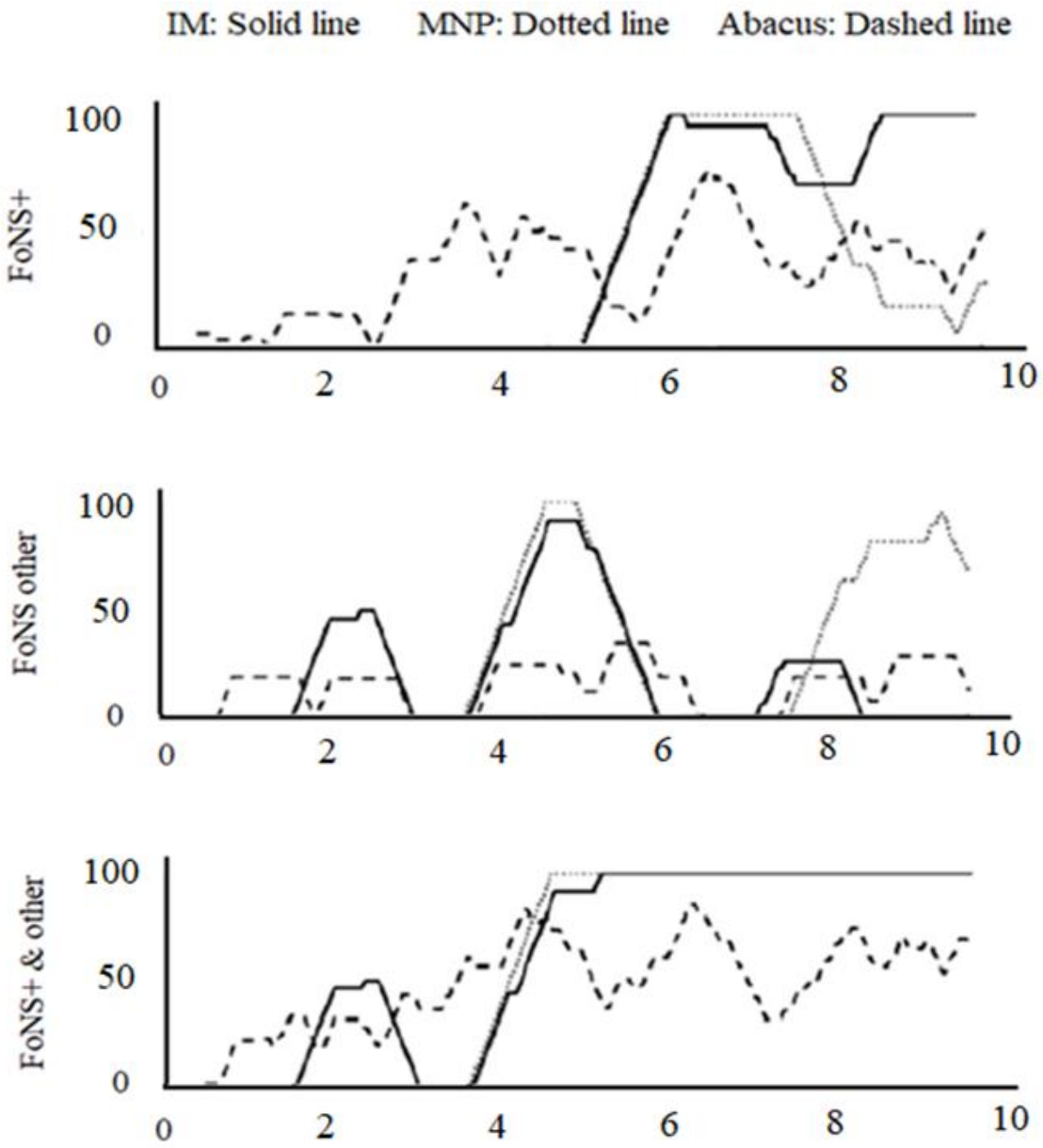

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What do these analyses tell us?

- Imported textbooks challenge local norms and expectations with respect to the nature and teaching of year-one mathematics.
- Singaporean imports do not address any FoNS category after the middle of the school year.
- The move away from number recognitions and counting strategies more rapidly than other textbooks.
- They shift to more challenging numbers more rapidly than other textbooks, particularly with respect to simple arithmetical operations
- They do not address number patterns, which is a core early competence
- In other words, Singaporean textbooks are not the solutions many would imagine them to be.

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In closing

- The act of importing textbooks is problematic
- Imported textbooks are written for particular curricula, and these may be very different from any receiving curricula.
- They represent different traditions and expectations that should not be assumed to match any receiving curricula.
- They represent, in frequently unacknowledge ways, different perspectives on the purpose of mathematics. For example,
- If I return to my starting problem, I know from my work with teachers in several countries, that importing a traditional Hungarian textbook would be a disaster.
- Culture influences what we teach, why we teach it and how we teach it.
- For example, returning to the EU-funded study...

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Teachers' observed didactics in whole class situations

	Flanders	England	Hungary	Spain
Activating prior knowledge	23	12	35	13
Exercising prior knowledge	3	8	5	0
Explaining	52	52	59	64
Sharing	61	60	97	61
Exploring	6	4	0	5
Coaching	39	54	45	76
Assessing or evaluating	20	14	36	1
Motivating	10	13	46	56
Questioning	49	6	87	71
Differentiating	6	8	0	4

The percentage of each country's episodes to which each code was applied

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