The Challenges of Mathematics Teaching and Learning (in the U.S.)

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Here is the U.S. version of the challenge:

Math is not most people's favorite subject…

Consider math anxiety ("about 93,800,000" Google hits in less than half a second)…

"Teen Talk Barbie": "math class is tough"…

Poor test scores; test performance gaps and racial/ethnic stereotypes…

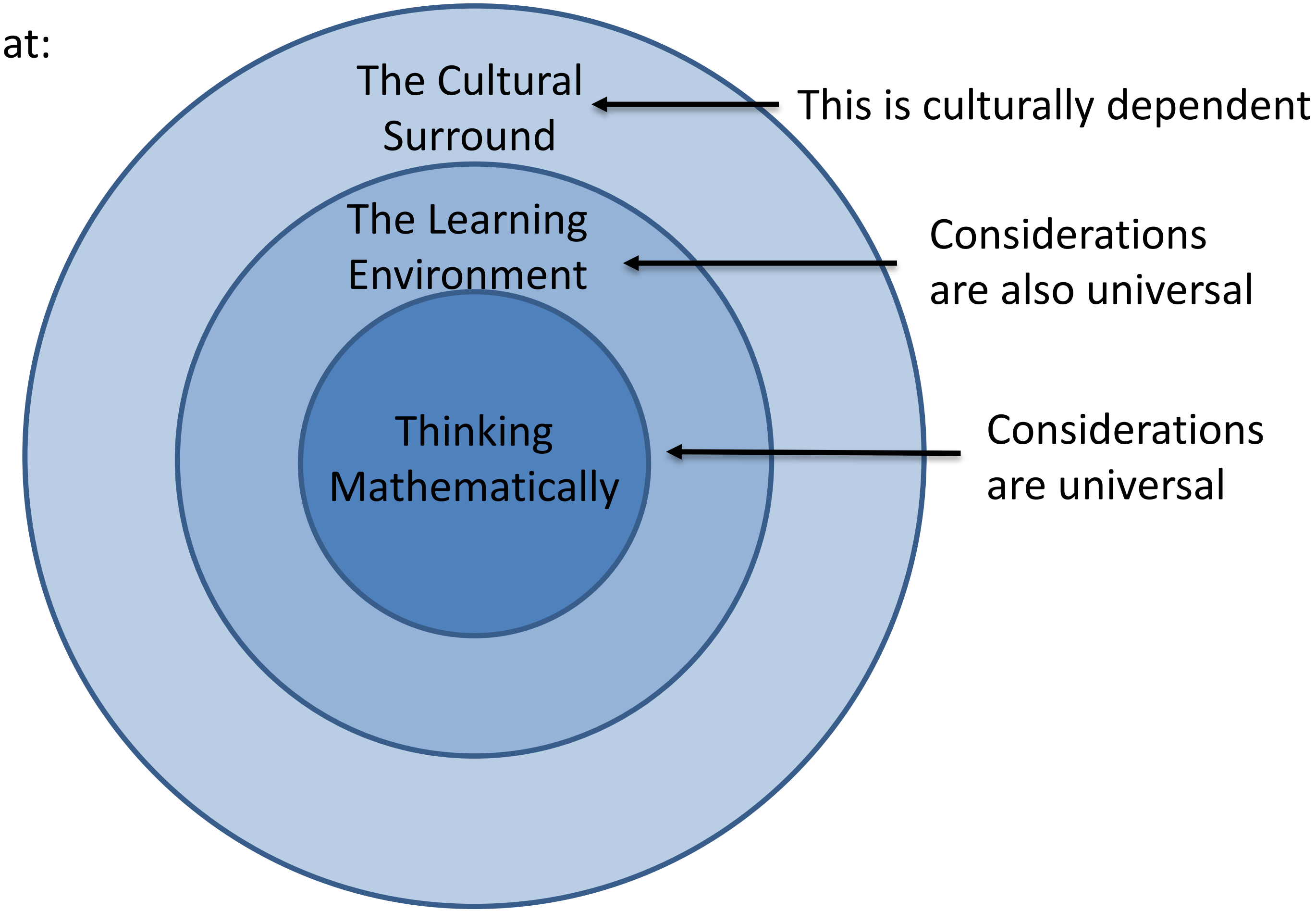
The belief that either you have the "math gene" or you don't…

Simply put, we have a math "problem."

(Tomas will discuss mathematics in Denmark…)

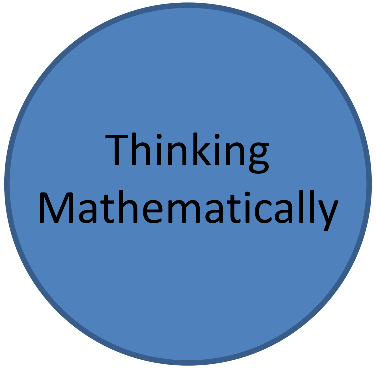
I will ague there are three interlocking aspects to the challenge…

I will also argue that:



Let’s theorize the space of inquiry,
from the inside out.

We'll start with the goal:



Just what's involved?

What determines your success or failure in "Thinking Mathematically" – Using mathematics effectively when it is called for?

(i) What you know and can do – your access to mathematical knowledge and practices

(ii) problem solving strategies (heuristics)

(iii) "control": monitoring and self-regulation, key parts of metacognition

(iv) Beliefs and dispositions – your sense of what doing math is, who you are mathematically (your identity), willingness to jump in (agency) and more.

We could spend weeks on this. Here are some 1-liners:

(i) What you know and can do – your access to mathematical knowledge and practices

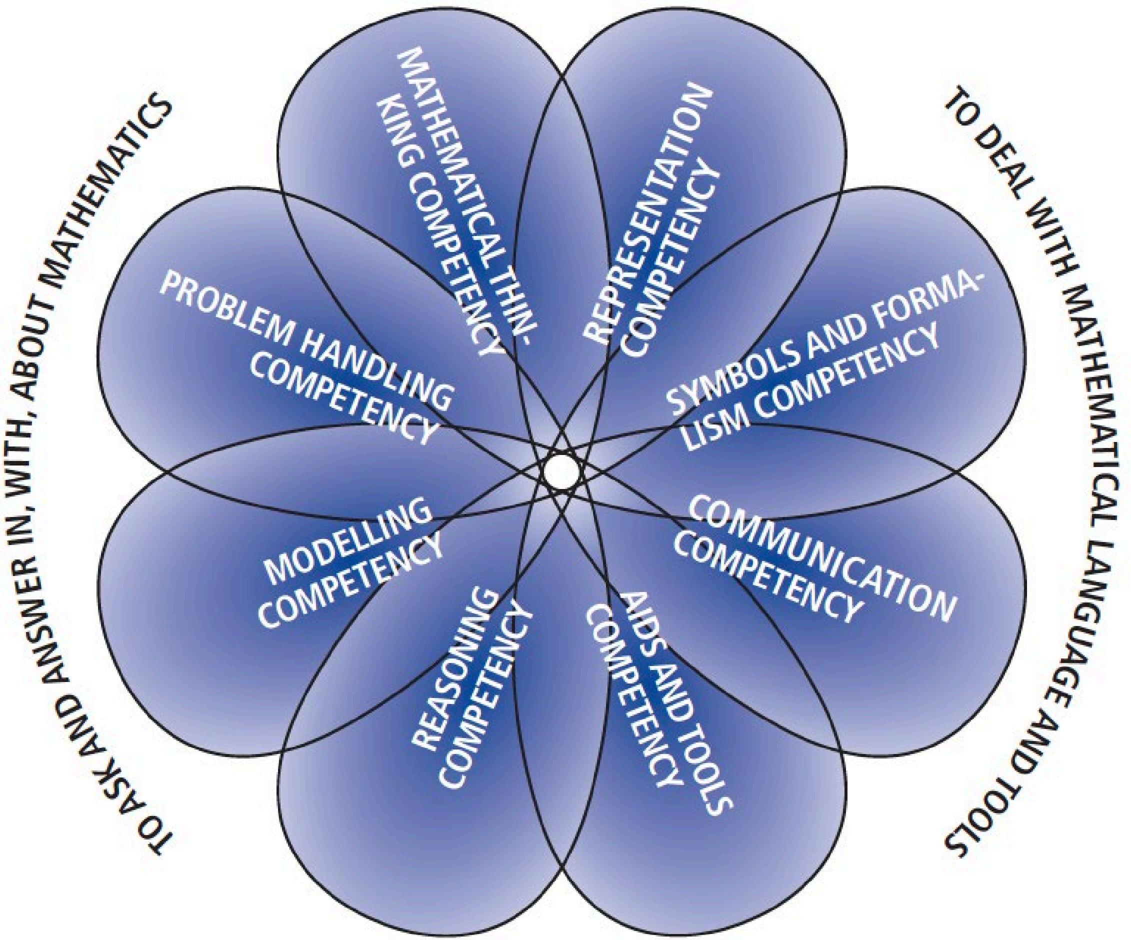
There's a lot more to this than meets the eye.

Knowledge is complex. Habits of mind such as

- looking for patterns, conjecturing, looking for counterexamples, proving;
- using multiple representations, making connections
- becoming a mathematical sense maker

require time and consistent experience to develop.

Of course, which mathematical content and practices students are held accountable to is a matter of national context. In the US, there are the "Common Core Standards;" in Denmark, this is more familiar:


TO ASK AND ANSWER IN, WITH, ABOUT MATHEMATICS
TO DEAL WITH MATHEMATICAL LANGUAGE AND TOOLS
MATHEMATICAL THINKING COMPETENCY
REPRESENTATION COMPETENCY
PROBLEM HANDLING COMPETENCY
SYMBOLS AND FORMALISM COMPETENCY
MODELLING COMPETENCY
COMMUNICATION COMPETENCY
REASONING COMPETENCY
AIDS AND TOOLS COMPETENCY

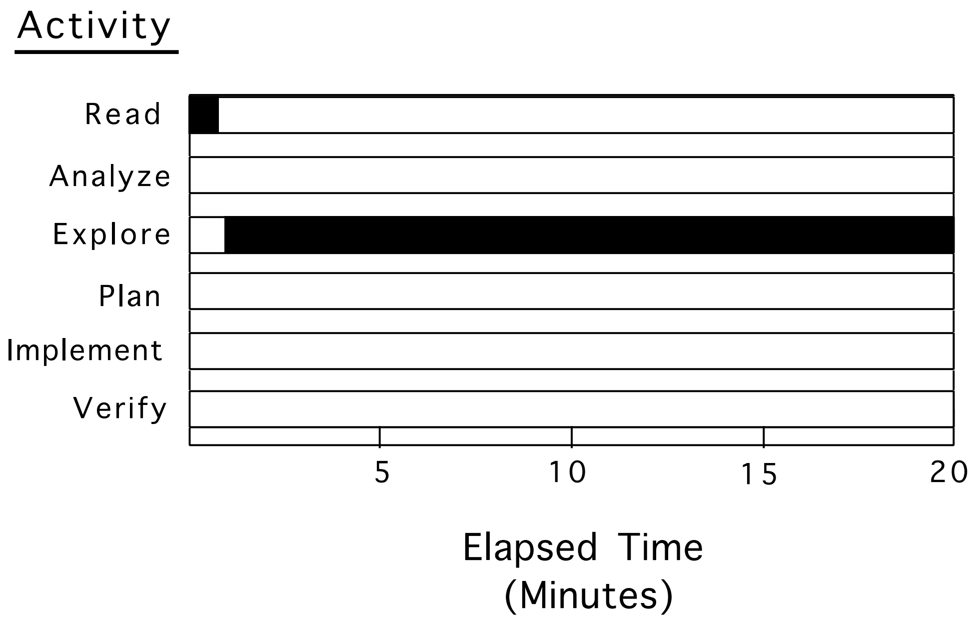
(ii) Problem solving strategies (heuristics)

The complexity of problem solving strategies has been tremendously underestimated. But the bottom line is that students *can* become effective problem solvers given the right opportunities.

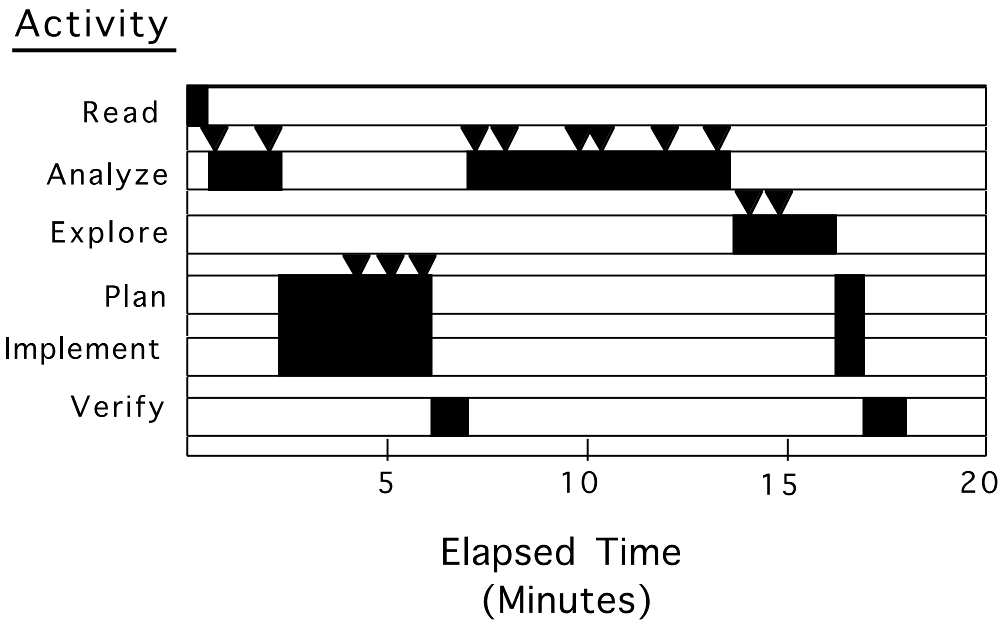
(Students in my problem solving classes solved problems I hadn't been able to solve.)

(iii) Control: monitoring and self-regulation

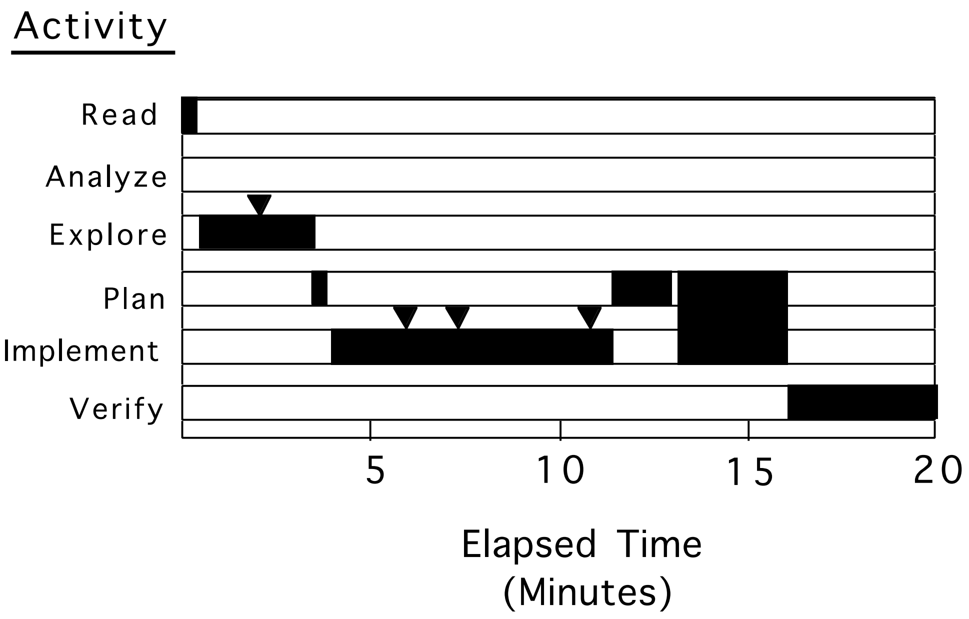
What follows are some timeline graphs indicating the seriousness of the issue…



Time-line graph of a typical student attempt to solve a non-standard problem.



Time-line graph of a mathematician
working a difficult problem



Time-line graph of two students working a problem after the problem solving course.

(iv) Beliefs and dispositions… Is it:

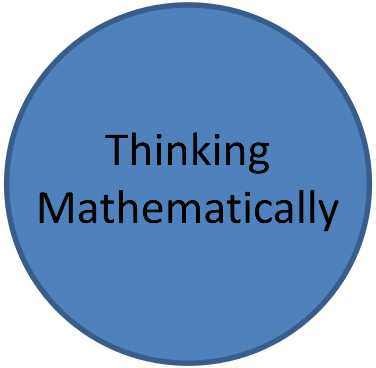
"Math is all memorizing" *versus* "Math is something you can figure out"?

"All problems can be solved in 5 minutes or less" *versus* "I can make progress over time"

"If I don't know how to do it, I'm dead" *versus* I'm willing to dig in and give it a shot."

These are issues of agency, authority, ownership, and identity.

All of these are aspects of

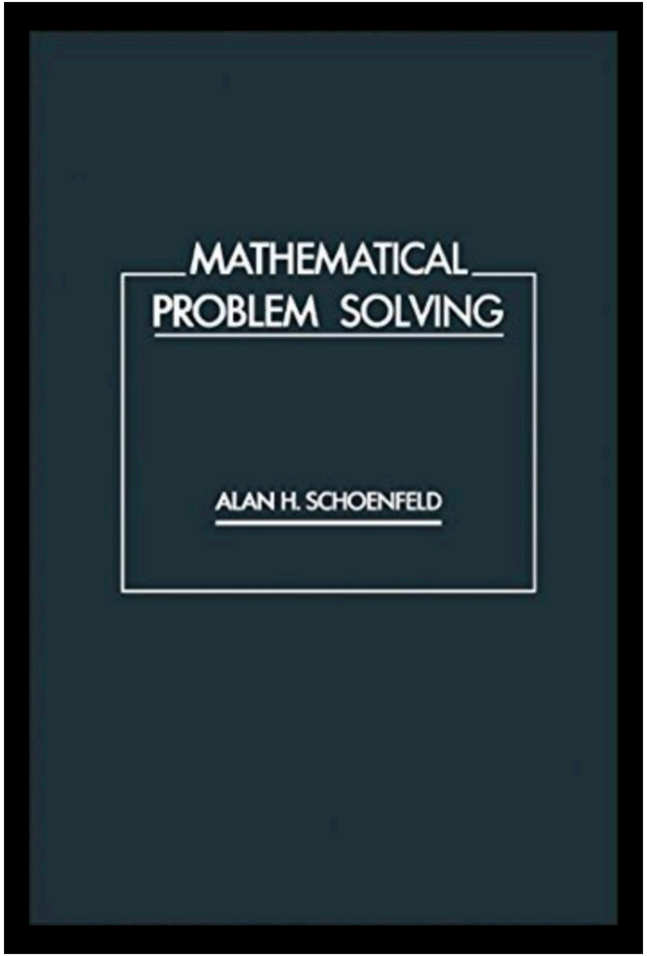


so we need to attend to them!

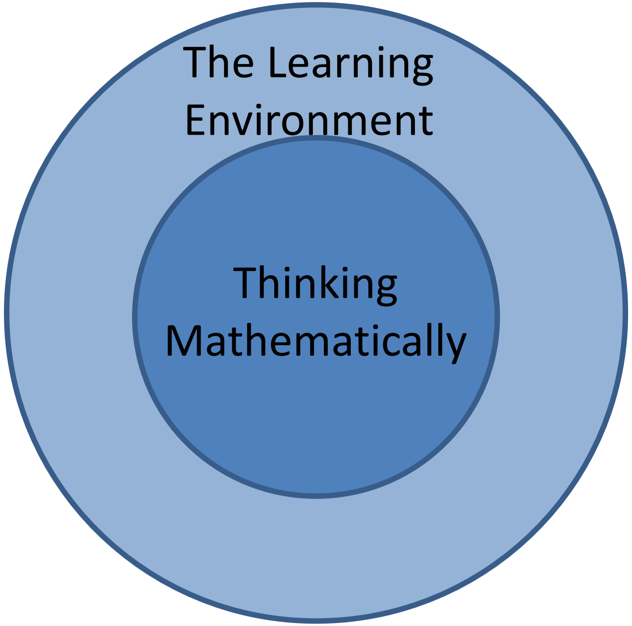
I believe this is the case in the US, in Denmark, everywhere.

(For a little light reading…)

The problem is:
we knew all this in
1985, and we're
STILL not doing
much of this in
the classroom!



And, of course, the place where learning to think mathematically typically occurs is the classroom…



so it's time to turn our attention to the essential properties of learning environments…

The Goal:

Understanding Classrooms that produce students who are Powerful Thinkers

(or not!)

Here's the big question:

What are 5 fundamental properties of classrooms from which students emerge as knowledgeable and resourceful thinkers and problem solvers?

You’re about to meet the
Teaching for **R**obust **U**nderstanding
(**TRU**)
Framework

I'm going to spare you the details of the 6 years of R&D that resulted in TRU …

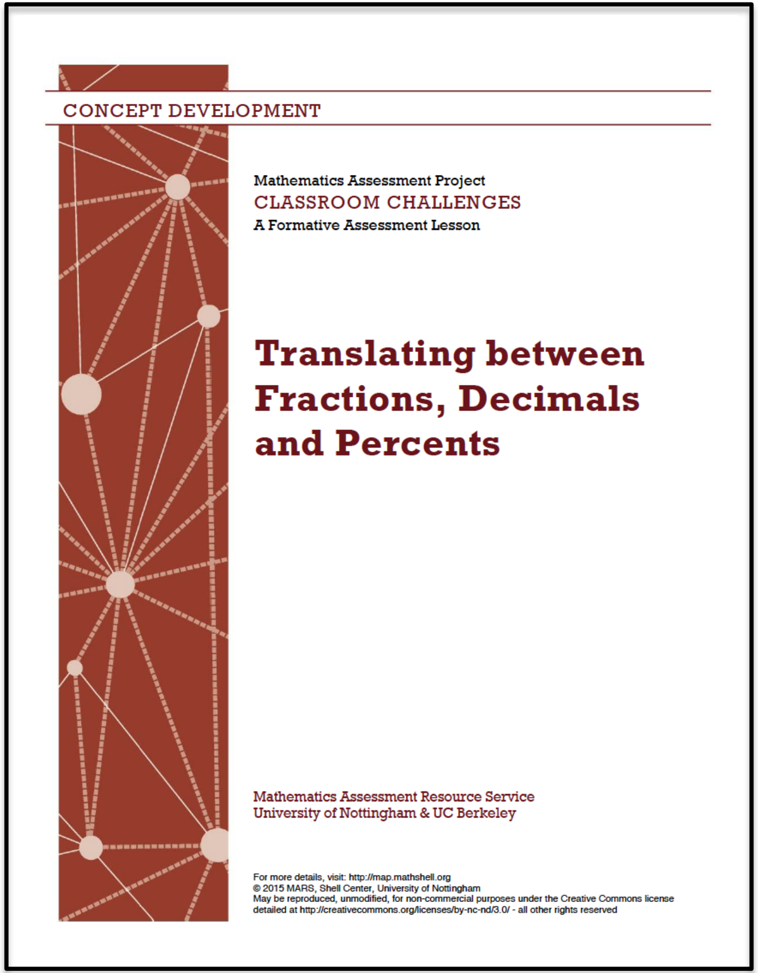
Say Thanks!*

… and instead show you just one video (sigh!) and use it to illustrate TRU *and* problematize looking at videos.

*Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM, 45: 607-621.*

The video:

6th grade Chicago students doing a "Formative Assessment Lesson" Titled "Translating Between Fractions, Decimals, and Percents."



CONCEPT DEVELOPMENT

Mathematics Assessment Project
CLASSROOM CHALLENGES
A Formative Assessment Lesson

Translating between Fractions, Decimals and Percents

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

For more details, visit: http://map.mathshell.org

This lesson is available for free, along with 99 other formative assessment lessons (a.k.a. “Classroom Challenges”). Just google “mathematics assessment” to find the Mathematics Assessment Project website.

To date we have over 7,500,000 lesson downloads.

Fractions, decimals, percents

Take turns to:

1. Fill in the missing decimals and percents.
2. Place the cards in order of size.
3. Check that you agree.

0.2 ___%	0.05 ___%	_.___ 80%
0.375 ___%	_.___ 12.5%	0.75 ___%
1.25 ___%	_.___ 50%	_.___ ___%

Fractions, decimals, percents

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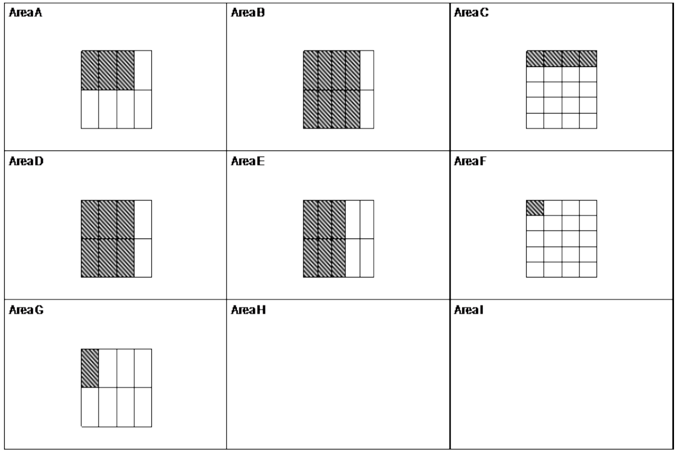
0.375 ___%	_.___ 12.5%	0.75 ___%
1.25 ___%	_.___ 50%	_.___ ___%

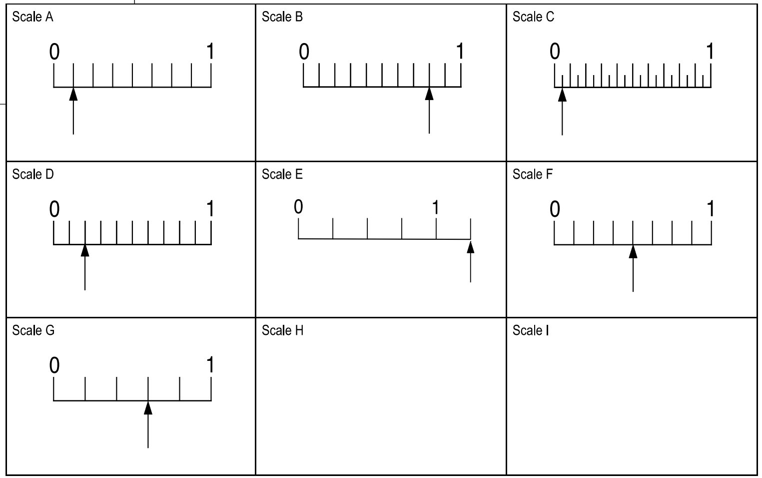


1. Match each area card to a decimals/percents card.
2. Create a new card or fill in spaces on cards until all the cards have a match.
3. Explain your thinking to your group.
4. Place your cards in order of size. Check that you all agree.

Fractions, decimals percents

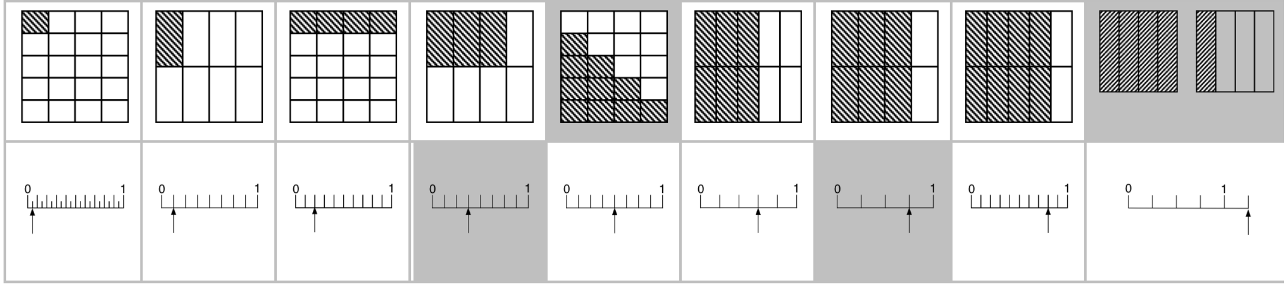
$\frac{3}{8}$	$\frac{4}{5}$	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{6}{10}$	$\frac{5}{4}$
$\frac{1}{8}$		

- Add these cards.
- Place all cards in order of size.
- Check that you agree.



Here’s the full solution.

0.05 5%	0.125 12.5%	0.2 20%	0.375 37.5%	0.5 50%	0.6 60%	0.75 75%	0.8 80%	1.25 125%
$\frac{1}{20}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$



The gray cards are the ones that students had to create for themselves.

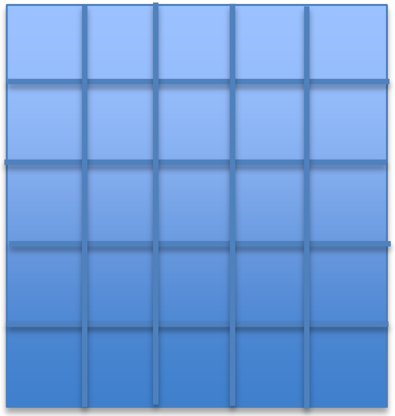
In this part of the lesson the students work in small groups. The teacher circulates.

In the first part of the clip one student is explaining to another how to convert 50% to a decimal.

Note how all the groups are explaining to each other.

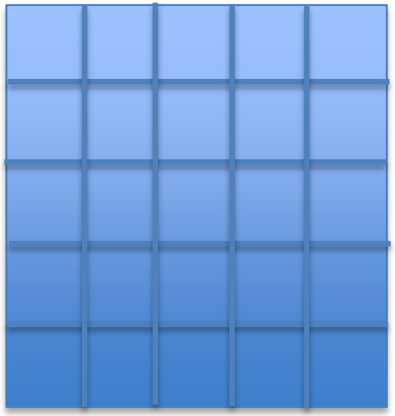
Later on, the students need to draw an area model and fraction for 1.25.

One student draws this:



A second student challenges him, saying he has shaded in 1, not 1.25.

The first student says no, each square is worth 5, so they add up to 125.



The second student leads the first through this argument:

You have a whole, which is 100%.

Which is bigger, 125% or 100%?

Doesn't that mean 125% should be bigger than a whole?

He leads the other student to see that 125% is 1 ¼.

Remember the big question:

What are 5 fundamental properties of classrooms from which students emerge as knowledgeable and resourceful thinkers and problem solvers?

Our distillation of the research, and a great deal of empirical work, suggests that the following five dimensions of classroom activity are essential.

I'll illustrate them by referring to the tape.

The Five Dimensions of Powerful (Mathematics) Classrooms

The Content	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful disciplinary thinkers. Discussions are focused and coherent, providing opportunities to learn disciplinary ideas, techniques, and perspectives, make connections, and develop productive disciplinary habits of mind.	*The extent to which students have opportunities to grapple with and make sense of important disciplinary ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core disciplinary content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations about disciplinary ideas, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Power ful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Note how this framework focuses on the learner's perspective.

Four of the five dimensions have to do with the ways in which the students experience the content.

What’s important about this framework?

Here are a few central points.

1. **The TRU Dimensions are necessary and sufficient. That is,**

If things go well along all 5 dimensions, students will emerge from the classroom being powerful thinkers.

If things go badly along *any* of the dimensions, they will not.

So, the key idea is to get better at all 5 dimensions.

2. TRU involves a fundamental shift in perspective, from teacher-centered to student-centered.

The key question is *not:* "Do I like what the teacher is doing?" It is:

"What does instruction feel like, from the point of view of the student?"

Note that four of the five TRU dimensions are about the student's experience of mathematics.

Observe the Lesson Through a Student's Eyes

The Content	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored? In what ways am I kept engaged?
Agency, Ownership, and Identity	• What opportunities do I have to explain my ideas? In what ways are they built on? • How am I recognized as being capable and able to contribute?
Formative Assessment	• How is my thinking included in classroom discussions? • Does instruction respond to my ideas and help me think more deeply?

3. TRU does not tell you how to teach, because there are many different ways to be an effective teacher.

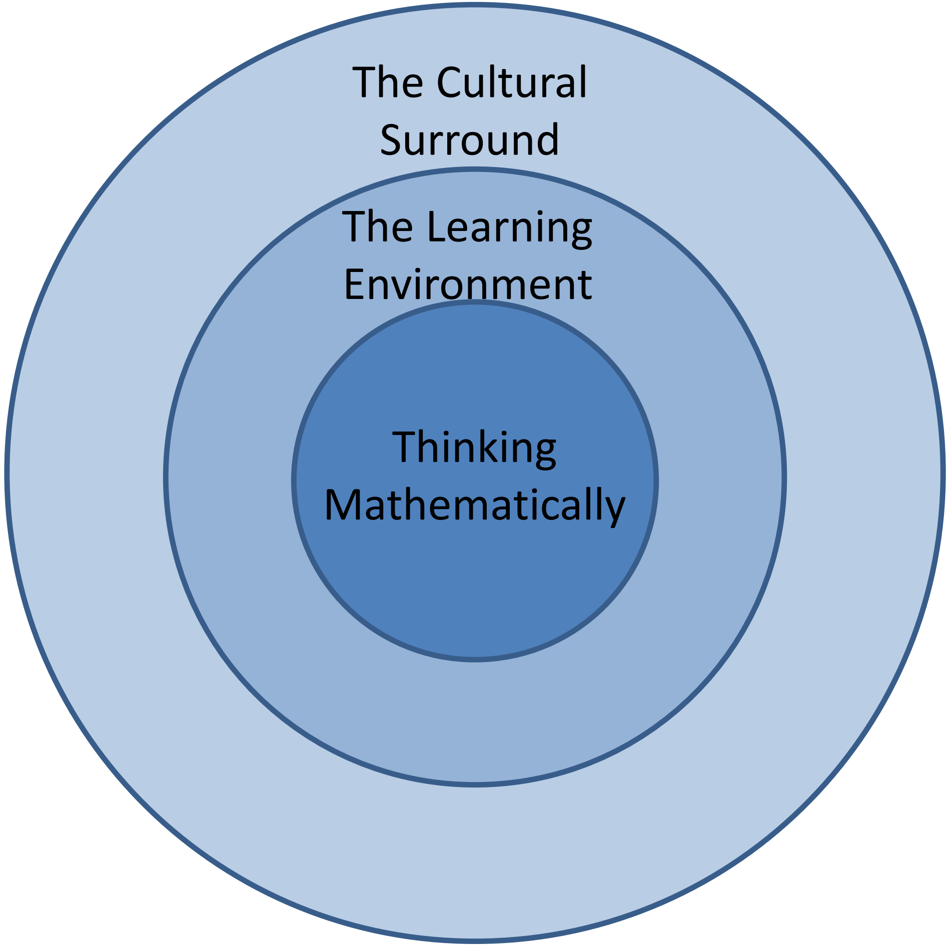
TRU describes the *principles* of powerful instruction, so it serves to *problematize* instruction. Asking, "how am I doing along this dimension; how can I improve?" makes your teaching richer, without telling you what to do.

You can find tons of detail about TRU at https://truframework.org/

And, we have two brand new books to help these ideas become a reality in the classroom:

I can’t guarantee, but I strongly believe that the ideas in TRU are universal.

Now, let’s turn explicitly to issues of context and the challenges they confront us with. Here my comments are very much about the U.S.


The Cultural Surround
The Learning Environment
Thinking Mathematically

Challenges related to Curricula in the U.S.

By and large, curricula are inadequate.

We know we can do better, it's a matter of investment.

The Formative Assessment Lessons are a case in point.

Challenges related to Assessment in the U.S.

In the US (and beyond), assessment stifles instruction – WYTIWIG.

It's not that we can't do it, we do know how. It's that the powers that be want cheap machine-gradable tests.

Challenges related to R&D in the U.S.

Hugh Burkhardt and I have written about this – the academic system offers negative incentives for doing the collaborative work that is necessary to make progress.

Challenges related to Teacher Support, 1

Covid-19 made it very clear that teachers and teaching are the lowest priority.

"Teachers Could Stay in Classroom if Exposed to COVID-19. New guidance from President Donald Trump's administration declares teachers to be 'critical infrastructure workers'."

Challenges related to Teacher Support, 2

Teacher “preparation” is generally inadequate.

Professional Development is laughable.

The mandate that teachers move to virtual instruction without support is an indication of the seriousness with with teaching is (not) taken.

Challenges related to Race and Racism in the U.S.

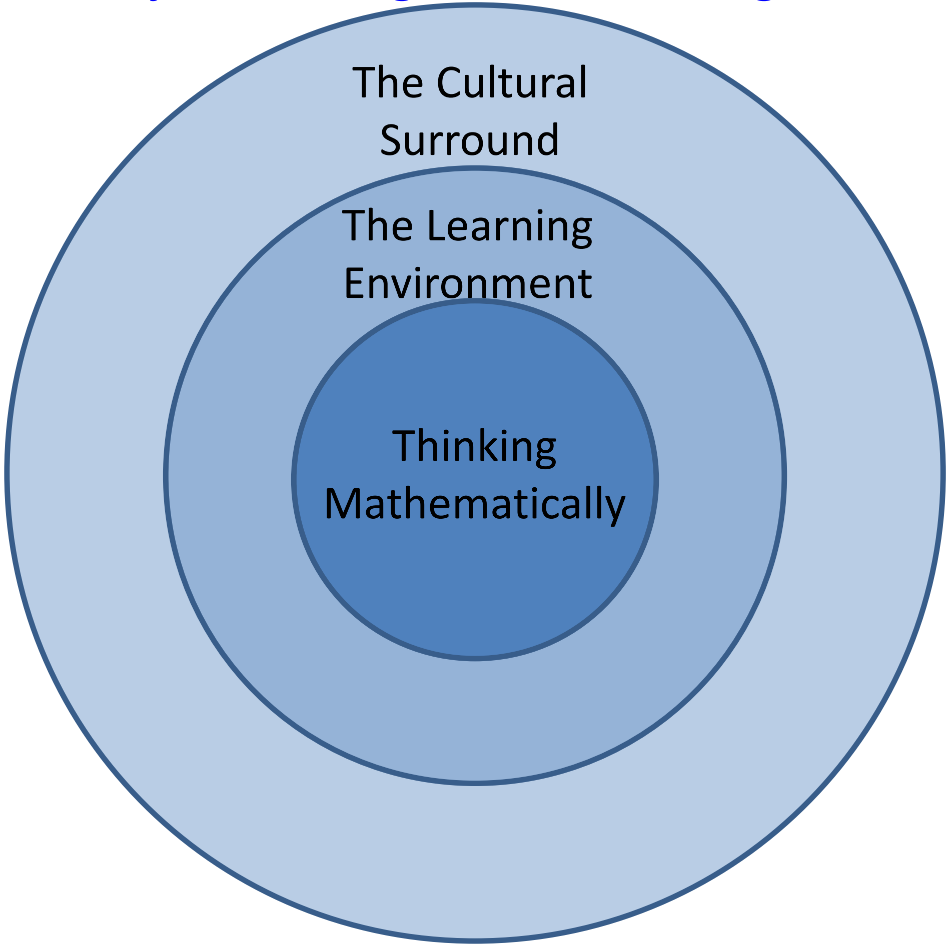
To mention just 3:

- Tracking
- “Savage inequalities”
- These issues permeate the classroom:

  “What are you doing here?”

So, to sum up…

There are major challenges in addressing each of these!



So, is it really surprising that mathematics teaching and learning are so challenging? (in the US, we'll see about Denmark).

This is what I say to my US colleagues:

There is a way out…

if we take stock and ask,

What really matters?

What should students learn?

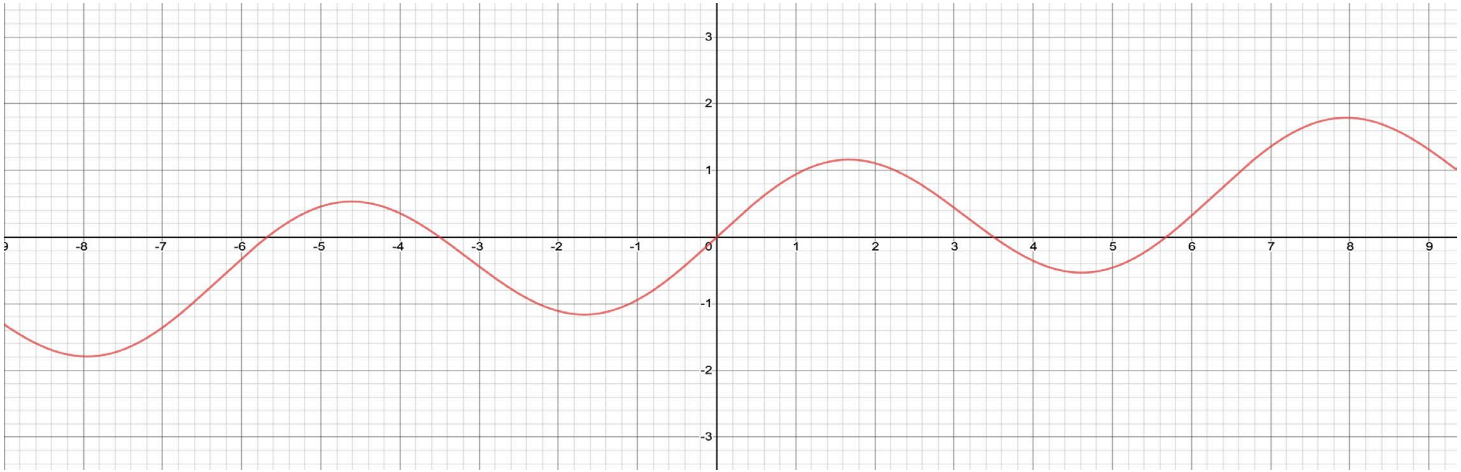
This could be the time to re-think mathematics education.

A final thought, since my students have said that the previous slides are too pessimistic:

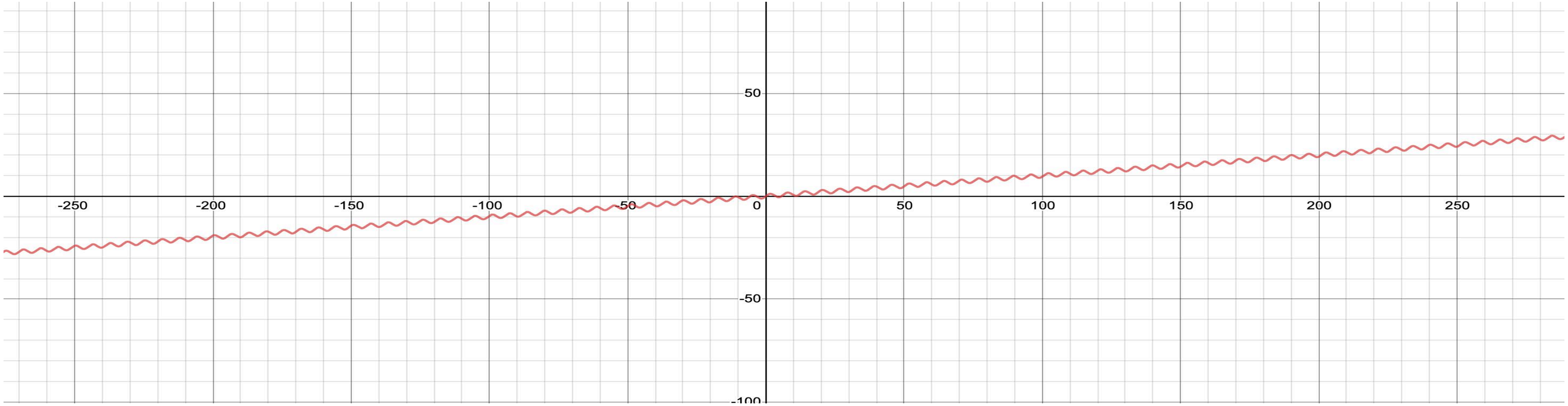
In my experience, progress looks like the equation

$$y = .1\,x + \sin x.$$

If you look at it over a short interval, it's like a roller coaster.



But if you take a distanced perspective, you can see the progress.



OK, I’m turning things over to Tomas
(And then to you.)

THANKS!