# Early Development of Number Knowledge: Identifying Risk of Learning Disability

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# OVERVIEW

- Focus on several key findings of a 4-year preschool to 1st grade longitudinal study
- Basic goal was to identify the core early quantitative knowledge that predicts readiness for math learning in 1st grade and risk of long-term learning difficulties
- First focus: Identify core risk areas.
- Second focus: Developmental patterns in these areas and subsequent mathematical development and achievement
- Third: Next steps, LD Innovation Hub

# Longitudinal Design

| Year | 3 yrs. | 4 yrs. | K | 1 |
|---|---|---|---|---|
| Math, Read Ach | | | | |
| IQ | | | | |
| Executive Control | | | | |
| Quantitative Tasks | | | | |
| Math Cognition | | | | |
| Work Memory | | | | |

# Sample

- **Characteristics**
  - **232 Title 1 preschool children**
  - **Dropped 14 for low IQ (< 70) and 21 for inattention or moved**
  - **Primary Sample; 197; IQ is average: *M* = 97, *SD* = 15**
  - **Mathematics achievement average; *M* = 95, *SD* = 15**
  - **52% girls, 55% white**
  - **38% very low income, 42% received food stamps, 9% housing**
  - **Started at  3 years, 9 months**

# Early Development of Number Knowledge

- **12 Quantitative Tasks Administered 4 times (twice per preschool year)**
  - **Bridge the gap between infancy/preschool number development and school-entry math competencies**
- **Core symbolic knowledge**
  - **Numeral identification; 1 to 15**
  - **Numeral comparison; which is larger '5' vs. '2'**
  - **Verbal counting; up to 100**
  - **Enumeration; counting set of objects while pointing at each**
  - **Cardinal knowledge; knowing the exact quantities represented by number words**
    - **Children learning 'one,' 'two,' 'three' individually and once they understand 'four' they generalize to larger count words (**Le Corre & Carey, 2007**), but don't induce that all words are *n*+1 for several more years (**Cheung et al., 2017**)**
    - **First conceptual understanding of symbolic mathematics**

# Early Development of Number Knowledge

- **Initial analyses:**
  - **Performance on an inherent system for representing quantity, approximate number system (ANS), predicts math achievement but is mediated by cardinal knowledge**
  - **Confirmed the mediation in follow-up analyses and through the end of preschool** (Chu et al., 2015)

- **In follow up, beginning of preschool quantitative predictors of end of preschool math achievement** (Geary & vanMarle, 2016)
  - **Series of Bayes regressions to identify best sets of domain general, non-symbolic, and symbolic predictors**
  - **Best in each set combined to reduce to final set of predictors, which were then used in standard regression**
  - **Cardinal principle knower (CPK) – contrasts of 6 knowers with 'one', 'two', 'three' and 'four' knowers**

# Early Development of Number Knowledge

## Predicting End of Preschool Mathematics Achievement

| Beginning of Preschool Variable | Estimates | t | p | Estimates | t | p |
|---|---|---|---|---|---|---|
| Intercept | .477 | 3.17 | .0018 | .434 | 2.55 | .0116 |
| Age | .083 | 1.58 | .1161 | .104 | 1.88 | .0615 |
| Girls contrasted with boys | -.189 | -1.88 | .0613 | -.148 | -1.40 | .1637 |
| No information contrasted with college | -.138 | -0.94 | .3470 | -.124 | -0.79 | .4300 |
| High school contrasted with college | -.107 | -0.90 | .3694 | -.111 | -0.88 | .3802 |
| Preliteracy: Letter Recognition | .370 | 6.28 | .0001 | --- | --- | --- |
| Nonverbal IQ | .149 | 2.74 | .0067 | .152 | 2.66 | .0086 |
| Numeral Recognition | --- | --- | --- | .302 | 4.13 | .0001 |
| One-knower contrasted with CP knower | -.575 | -3.11 | .0022 | -.621 | -3.11 | .0022 |
| Two-knower contrasted with CP knower | -.408 | -2.50 | .0134 | -.372 | -2.02 | .0448 |
| Three-knower contrasted with CP knower | -.276 | -1.48 | .1399 | -.203 | -0.99 | .3245 |
| Four-knower contrasted with CP knower | -.276 | -1.61 | .1085 | -.205 | -1.07 | .2875 |
| Verbal Counting | .202 | 3.37 | .0009 | .182 | 2.81 | .0055 |
| Ordinal Comparison | .108 | 2.13 | .0343 | .120 | 2.23 | .0266 |

# Early Development of Number Knowledge

- **What's the relation between the age of becoming a cardinal principle knower and later achievement** (Geary et al., 2018)**?**
  - **Is knowing this by kindergarten entry sufficient, or the earlier the better?**

- **141 of the 197 children completed assessments through 1st grade**
  - **Included quantitative tasks that predicted end of preschool math achievement and ANS acuity** (Geary & vanMarle, 2016)
  - **Number processing, addition strategy, and number line tasks that formed a Number System Knowledge (NSK) composite**
  - **NSK at beginning of 1st grade predicted employment-relevant math competencies in adolescence better than math achievement tests** (Geary et al., 2013)

| Measure | Mean Age |
|---|---|
| **First Year of Preschool** | |
| Give-a-Number | 3y 10m |
| Discrete Quantity Discrimination | |
| Object Comparison | |
| Verbal Counting | 3y 11m |
| Numeral Recognition | |
| Executive Functions | 4y 1m |
| WIPPSI | |
| Letter Identification | |
| Give-a-Number | 4y 2m |
| **Second Year of Preschool** | |
| Give-a-Number | 4y 9m |
| Executive Functions | 5y 0m |
| Give-a-Number | 5y 2m |
| Discrete Quantity Discrimination | |
| Object Comparison | |
| Verbal Counting | 5y 2m |
| Numeral Recognition | |
| Test of Early Mathematical Abilities | 5y 4m |
| **Kindergarten** | |
| Numerical Operations | 6y 2m |
| Word Reading | |
| Coloured Progressive Matrices | |
| **First Grade** | |
| Addition Strategy | 6y 9m |
| Number Sets | |
| Number Line | |

# Early Development of Number Knowledge

- **Used combination of Bayes and frequentist statistics to identify the best preschool predictors of NSK**
  - **Age of becoming CPK (GiveN > 4) contrasted with children who did not achieve it by end of preschool**
  - **For discrete quantity discrimination (ANS), object comparison, verbal counting, numeral recognition, used start of preschool scores and gains from beginning to end of preschool as predictors**
  - **Domain-general predictors: executive function in first and second year of preschool, preschool IQ, preschool letter knowledge, and kindergarten IQ**
  - **Best set of predictors in Bayes regressions used in standard regressions**
  - **Added control of end of preschool mathematics achievement**
  - **Then, all predictors**
  - **Follow-up prediction of end of kindergarten Math and Reading Achievement**

# Early Development of Number Knowledge

## Predicting Beginning of 1st Grade Number System Knowledge

| Predictor | Estimates | t | p | Estimates | t | p | Estimates | t | p |
|---|---|---|---|---|---|---|---|---|---|
| Intercept | -.6808 | -3.13 | .0022 | -.5763 | -2.71 | .0076 | -.4893 | -2.00 | .0474 |
| Parental Ed: No Information vs. College | ---- | ---- | ---- | ---- | ---- | ---- | -.0889 | -0.48 | .6302 |
| Parental Ed: High School vs. College | ---- | ---- | ---- | ---- | ---- | ---- | -.2508 | -1.86 | .0648 |
| Letter Recognition | ---- | ---- | ---- | ---- | ---- | ---- | -.1579 | -1.77 | .0788 |
| Nonverbal Intelligence | ---- | ---- | ---- | ---- | ---- | ---- | -.0135 | -0.21 | .8322 |
| Verbal Intelligence | ---- | ---- | ---- | ---- | ---- | ---- | -.0497 | -0.75 | .4565 |
| Executive Functions: Year 1 | ---- | ---- | ---- | ---- | ---- | ---- | -.0115 | -0.17 | .8687 |
| Executive Functions: Year 2 | ---- | ---- | ---- | ---- | ---- | ---- | .1016 | 1.47 | .1452 |
| End of Preschool Math Achievement | ---- | ---- | ---- | .2767 | 3.24 | .0015 | .2503 | 2.85 | .0052 |
| Progressive Matrices | .3064 | 4.51 | .0001 | .2389 | 3.47 | .0007 | .2559 | 3.56 | .0005 |
| Cardinal Principle Knower by 3y 10m | 1.071 | 3.94 | .0001 | .9161 | 3.43 | .0008 | .9627 | 3.21 | .0017 |
| Cardinal Principle Knower by 4y 2m | .9854 | 3.57 | .0005 | .8279 | 3.06 | .0027 | .9105 | 3.10 | .0024 |
| Cardinal Principle Knower by 4y 9m | .5781 | 2.41 | .0173 | .4809 | 2.06 | .0415 | .5192 | 2.13 | .0350 |
| Cardinal Principle Knower by 5y 2m | .0945 | 0.36 | .7167 | .0873 | 0.35 | .7284 | .1202 | 0.47 | .6395 |
| Beginning Numeral Recognition | .2302 | 2.46 | .0151 | .1004 | 1.02 | .3111 | .1863 | 1.59 | .1148 |
| Gains in Numeral Recognition | .1720 | 2.11 | .0368 | .0618 | 0.72 | .4724 | .0891 | 1.02 | .3104 |

# Early Development of Number Knowledge

| | Word Reading | | | Numerical Operations | | |
|---|---|---|---|---|---|---|
| Predictor | Estimates | t | p | Estimates | t | p |
| Intercept | -.1591 | -0.61 | .5405 | -.3855 | -1.30 | .1965 |
| Parental Ed: No Information vs. College | .2901 | 1.49 | .1398 | -.1492 | -0.67 | .5058 |
| Parental Ed: High School vs. College | .2291 | 1.60 | .1111 | -.1180 | -0.72 | .4719 |
| Letter Recognition | .2096 | 2.22 | .0283 | .0689 | 0.64 | .5257 |
| Nonverbal Intelligence | .0324 | 0.48 | .6306 | .0208 | 0.27 | .7869 |
| Verbal Intelligence | .1038 | 1.47 | .1437 | -.0428 | -0.53 | .5975 |
| Executive Functions: Year 1 | .0021 | 0.03 | .9777 | -.0065 | -0.08 | .9383 |
| Executive Functions: Year 2 | .0928 | 1.26 | .2092 | .0367 | 0.44 | .6640 |
| End of Preschool Math Achievement | .4385 | 4.70 | .0001 | .0520 | 0.49 | .6271 |
| Progressive Matrices | .0747 | 0.98 | .3296 | .2751 | 3.15 | .0021 |
| Cardinal Principle Knower by 3y 10m | .0634 | 0.20 | .8422 | .8679 | 2.38 | .0186 |
| Cardinal Principle Knower by 4y 2m | -.0415 | -0.13 | .8943 | .4199 | 1.18 | .2418 |
| Cardinal Principle Knower by 4y 9m | .0317 | 0.12 | .9024 | .4089 | 1.38 | .1693 |
| Cardinal Principle Knower by 5y 2m | -.1531 | -0.56 | .5739 | .0983 | 0.32 | .7525 |
| Beginning Numeral Recognition | .0875 | 0.70 | .4830 | .0931 | 0.65 | .5146 |
| Gains in Numeral Recognition | .0268 | 0.29 | .7731 | .1053 | 0.99 | .3239 |

# Early Development of Number Knowledge

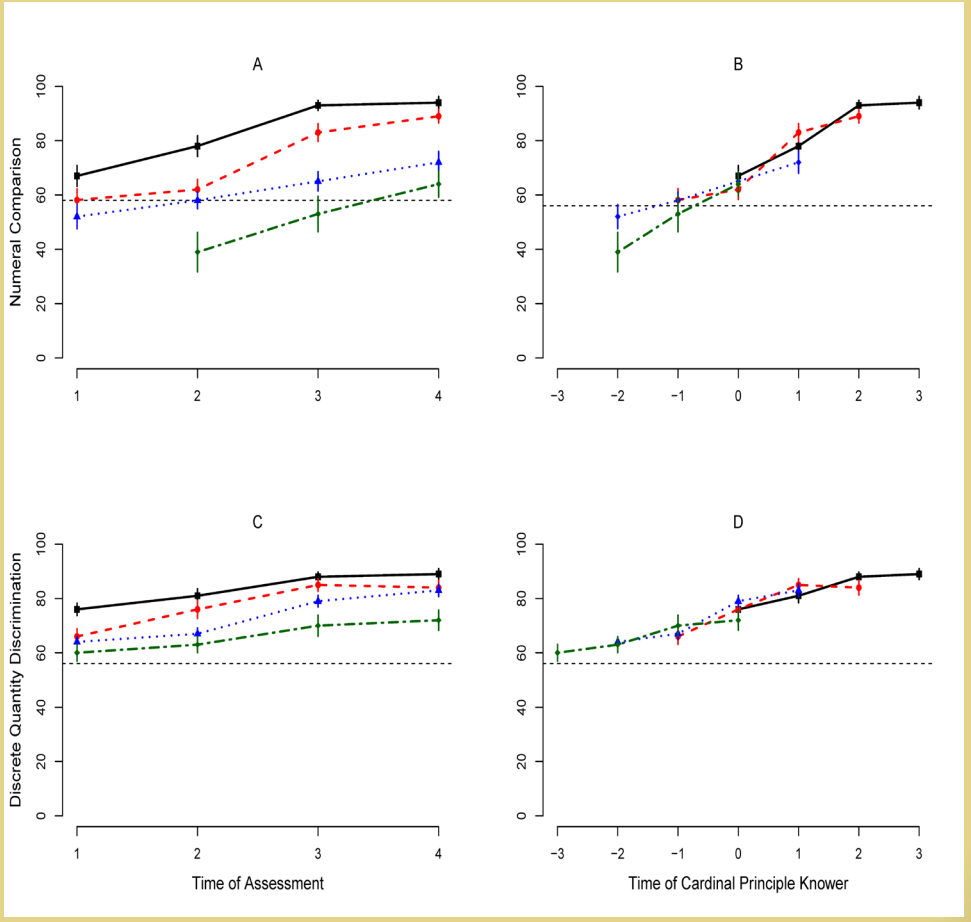
- **Core results:**
  - **Children who are cardinal principle knowers (CPKs) by about 4 years – before starting or early in the first year of preschool – have a 1 *SD* advantage on NSK in 1st grade, controlling IQ, prior achievement, executive function, and parental educational background**
  - **Age of CPK was unrelated to later reading achievement, demonstrating discriminant validity, but was related to later math achievement, although less strongly than to NSK**

- **Implication: Children's understanding of symbolic mathematics accelerates after becoming CPK**

- **In follow up study, we examined the relation between age of CPK and numeral comparison** (Geary & vanMarle, 2018)
  - **In older children, speed of numeral comparisons is a stronger predictor of concurrent and later math achievement than ANS acuity** (e.g., De Smedt et al., 2013)
  - **Numeral comparison is early stage of learning relations among numbers**

# Early Development of Number Knowledge

- **Approach:**
  - **Children were randomly presented with pairs of numerals (1 to 15) they recognized and asked to identify the larger one**
  - **Examined Numeral Comparison accuracy and ANS (discrete quantity discrimination) accuracy based on assessment time and then aligned to age of becoming a CPK**
- **Follow up: ANS and Numeral Comparison scores were correlated only after children became CPK**
  - **ANS representations become integrated with symbolic knowledge**

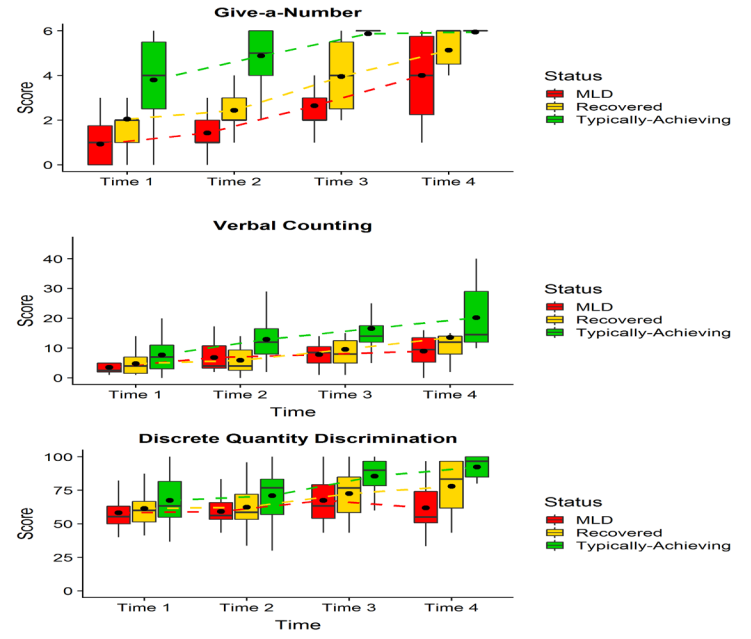



Accuracy for performance on the numeral comparison and discrete quantity discrimination tasks for groups that become cardinal principle knowers at time 1(black, solid), time 2 (red, dashed), time 3 (blue, dot), and time 4 (green, two dashed)

# Early Development of Number Knowledge

- **What happens with children at high risk of math learning disability at the end of preschool** (Chu et al., 2019)**?**
  - **At-risk < 25 percentile in both preschool years**
  - **Typically-achieving had achievement scores in the average range for both preschool assessments and at end of 1st grade**
- **Unexpected – many of the at-risk students recovered:**
- **Recovered students:**
  - **Substantive gains in EF:**
  - **Grains in Cardinal knowledge and CPK**
- **Still had subtle NSK deficits**

| | Math Learning Disability (n = 14) | | Recovered (n = 23) | | Typically-Achieving (n = 35) | |
|---|---|---|---|---|---|---|
| Math Achievement | Mean | %ile | Mean | %ile | Mean | %ile |
| Preschool Yr. 1 | 77[a] | 7 | 79[a] | 10 | 98[b] | 46 |
| Preschool Yr. 2 | 79[a] | 10 | 81 | 12 | 99[b] | 48 |
| End of 1st Grade | 78[a] | 8 | 100[b] | 49 | 100[b] | 50 |
| Executive Function | | | | | | |
| Preschool Yr. 1 | 22 | -- | 26 | -- | 32 | -- |
| Preschool Yr. 2 | 34 | -- | 44 | -- | 44 | -- |



# Early Development of Number Knowledge: Multisystemic Approach

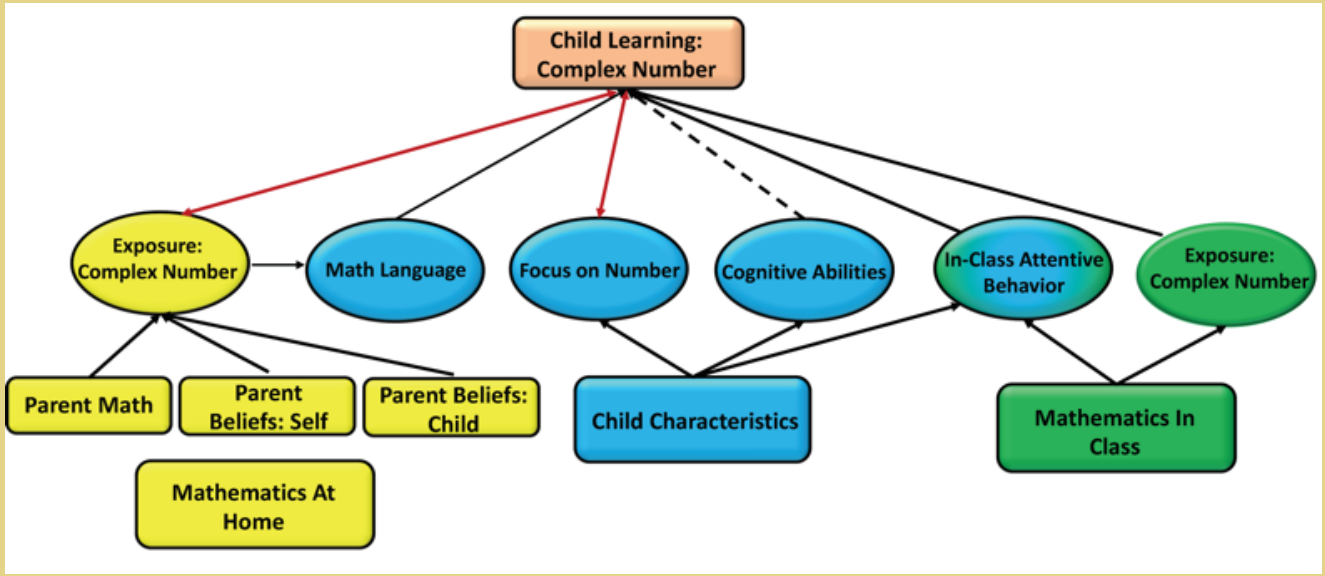
- **Core finding: Cardinal knowledge is the linchpin to early math development and age of acquiring this knowledge is critical:**
  - **At least a 2-year gap between early and late CPK. Why?**
- **LD Hub takes a multi-systemic approach focused on cardinal knowledge and supporting number competencies (e.g., enumeration) focusing on *child-centered*, *home-centered*, and *classroom-centered* factors and their interactions that could influence this early number development.**
- **Goal: 150 Children and their Families from same Title I Program as prior study.**

# Early Development of Number Knowledge: Multisystemic Approach

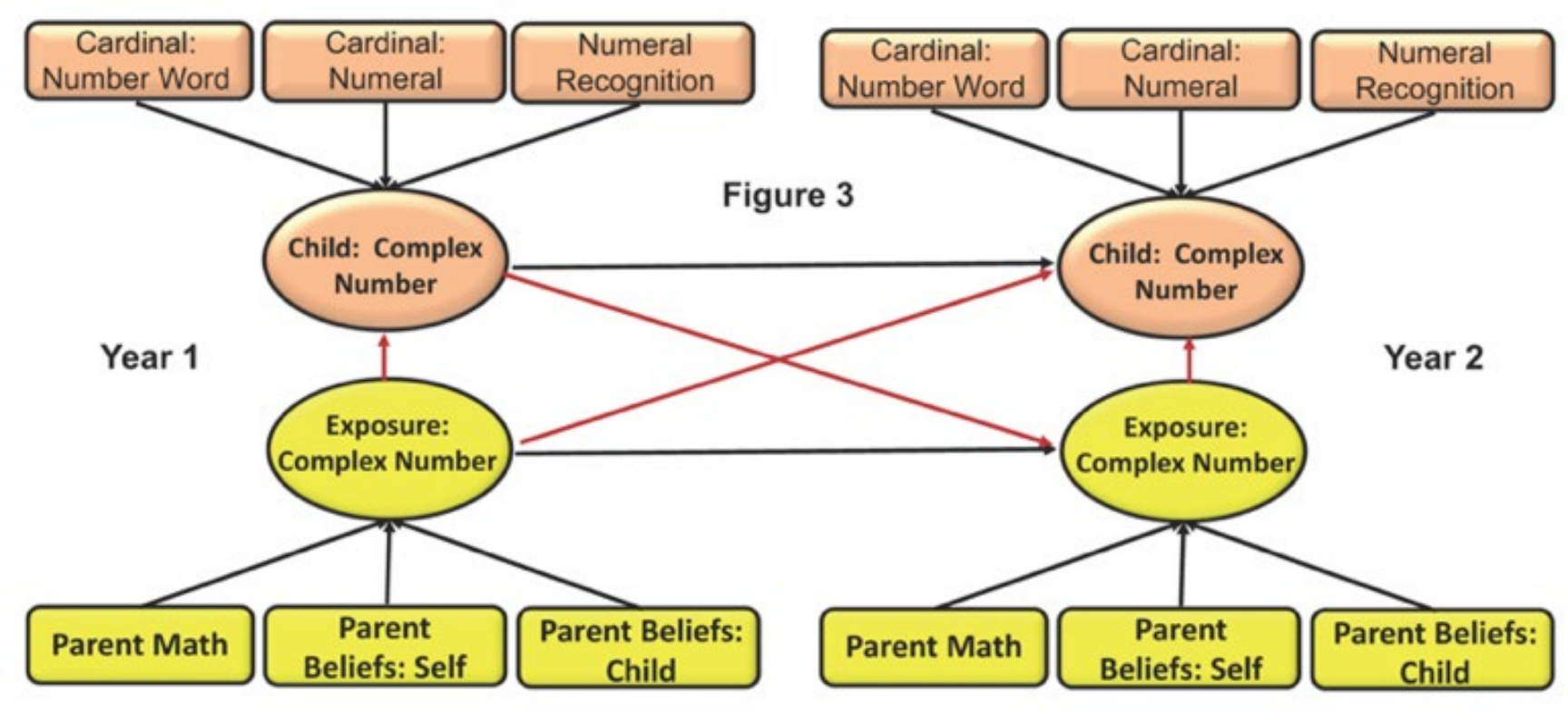
| Measures | Y1 Fall | Y1 Spr | Y2 Fall | Y2 Spr |
|---|---|---|---|---|
| **CHILD** | | | | |
| ***Quantitative Tasks*** | | | | |
| 1. Spontaneous focus number | √ | √ | √ | √ |
| 2. Verbal counting | √ | √ | √ | √ |
| 3. Enumeration | √ | √ | √ | √ |
| 4. Cardinal Knowledge: Number words | √ | √ | √ | √ |
| 5. Cardinal Knowledge: Arabic numerals | √ | √ | √ | √ |
| 6. Numeral recognition | √ | √ | √ | √ |
| 8. Numeral comparison | √ | √ | √ | √ |
| 9. Ordinal Choice (intuitive understand more, less) | √ | √ | √ | √ |
| 10. Math Vocabulary | √ | | √ | |
| 11. Number Line | | | | √ |
| ***Domain General*** | | | | |
| 1. Executive Function | √ | | √ | |
| 2. Letter Identification | √ | | √ | |
| 3. Wechsler IQ & WM | √ | | | |
| 4. Math Achievement TEMA | | √ | | √ |
| **CLASSROOM** | | | | |
| 1. In-class Attention: Teacher | | √ | | √ |
| 2. In-class Attention: Observation | | √ | | √ |
| 3. Opportunities to Learn: Teacher Rpt & Observation | √ | √ | √ | √ |
| **HOME** | | | | |
| ***Parent*** | | | | |
| 1. Math Anxiety & Attitudes (self and children) | | √ | | |
| 2. Cognitive abilities | | √ | | |
| 3. Math & Read Achieve | | √ | | |
| 4. Demographics | √ | | √ | |
| ***Home Environment*** | | | | |
| 1. Home Learning Opportunities (e.g., number bks) | √ | | √ | |
| 2. HOME Assessment | √ | | √ | |
| 3. Parent-Child Number Talk | √ | | √ | |

# Early Development of Number Knowledge: Multisystemic Approach

- **Basic Model: Complex Number Development related to:**
  - **Mathematical Home Environment**
  - **Child Characteristics**
  - **Classroom Experiences**
- **Assume Developmental Interactions (red lines)**
  - **Children's number knowledge might influence complexity of parent-child number talk that in turn enhances number development**



# Early Development of Number Knowledge: Multisystemic Approach



# Conclusion

- **Although an inherent sensitivity to quantity is correlated to math achievement, it does not appear to be central to later learning**
- **Rather, children's understanding of cardinality appears to be the key** (Carey, 2004).
  - **Unique to humans**
  - **It is their first mathematical induction, although continues to be elaborated for several years**
- **Learning cardinal value of individual words and numerals is the first step in learning relations among number symbols and the number system knowledge that predicts longer-term outcomes** (Geary et al., 2013)
- **One critical finding is that individual differences at the beginning of preschool are related to trajectory of number learning and readiness for school math.**
- **The factors contributing to early variation in number knowledge are not well understood**
  - **The LD Hub is focused on identifying these factors**
  - **These will be targets for a multisystemic intervention**