Programme & Abstracts

Welcome to NORSMA 11

22 - 24 November 2023
Copenhagen · Denmark

www.conferences.au.dk/norsma

NORSMA 11
Nordic Research Network on Special Needs Education in Mathematics

NCUM
Nationalt Center for Udvikling af Matematikundervisning

Index

NORSMA 11
Nordic Research Network on Special Needs Education in Mathematics


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Welcome to NORSMA 11

On behalf of the Nordic committee for NORSMA 11, it gives me great pleasure to welcome you to the **Nordic Research Conference on Special Needs Education in Mathematics**.

This year is the eleventh time, and this time Denmark has had the honor of hosting the event which we look forward to carrying out.

The conference will give you the opportunity to meet interesting people from other countries and other regions. Meetings which may develop into subsequent contacts and network formation as well as provide an opportunity to discuss and exchange experiences on central issues about special education in mathematics. Especially our conference dinner on Thursday evening will create good opportunities to talk, discuss and network. Through the various sessions, the conference will give you an insight into the research and development projects that have been completed or are still ongoing in the Nordic countries in recent years. In this context, it is planned to publish a special issue of the Danish magazine MONA with the presentations from the individual sessions.

We have also invited central international researchers as keynote speakers who represent different perspectives on difficulties in mathematics. Presentations that we hope you will find relevant and interesting. These presentations will be filmed and subsequently posted on the website.

The planning has been going on for a long time, and I must initially thank the Danish local committee for a multi-year collaboration with representatives from various educations and associations.
I would also like to thank NCUM (the (Danish) National Center for the Development of Mathematics Education) which is the organisation that forms the framework for the conference. In addition, the association DANSMA (Danish Special Mathematics), the university colleges KP and VIA and DPU Aarhus University have contributed to the planning of the conference. DPU Copenhagen Campus has also contributed significantly to the planning and hosts the conference.

We hope you will enjoy the conference and your time in Copenhagen.

On behalf of the Local Organising Committee and NORSMA, we give our warmest welcome to Copenhagen and NORSMA.

Sincerely,
Bent Lindhardt, Conference Chair

General Information

Wifi
Network name:
AU-Guest
(no password needed)

Registration opening hours
22 November: 8.30-16.30
23 November: 8.30-16.30
24 November: 8.30-15.30

Certificates of attendance
After the conference, you will receive an email from the Conference Secretariat with a link to download your certificate of attendance.

Presentation slides
We will collect all the slides from presentations and workshops at both the Teacher's Day and NORSMA 11 and make them available to conference participants after the conference.

Submit your abstract for publication in MONA
All presenters at NORSMA 11 are invited to submit a paper based on their abstract for publication in a special edition of MONA. Papers should be max. 10 pages incl. references and the submission deadline is 15 March. For more information, please visit www.conferences.au.dk/norsma

Speaker information
When speaking in the plenary room:
Please upload your presentation on the pc in the plenary room (Festsalen) in the break before the session in which you are presenting (at the latest). If you are presenting in one of the morning sessions on Thursday or Friday, please upload your presentation in the afternoon the day before you are presenting.
An assistant will be there to help you transfer the presentation to the computer.
Please bring your presentation on a USB stick.

When speaking in the smaller rooms:
Please bring your own computer and use this for your presentation.
The room will be equipped with a screen and a projector – for the larger rooms, there is also a sound system and microphones.

NCUM Annual Conference/ Teacher's Day

See the detailed programme with workshop descriptions at www.conferences.au.dk/norsma

Wednesday, 22 November

Time	
09.30	**Coffee & Registration**
10.00	**Welcome**
10.10	**NCUM's communication in the perspective of the conference** Morten Blomhøj, Director of NCUM and Professor, DPU
10.30	**Effective teaching strategies for dyscalculia and learning difficulties in mathematics** Marie-Pascale Noël, Professor, Psychological Sciences Research Institute, University of Louvain, Belgium
11.30	**Discussion and Q&A**
12.00	**Lunch**
13.00	**Workshops 1 - 8**
15.00	**Break**
15.15	**How can it be ensured that special efforts for students with mathematics difficulties has an effect in the long term?** Anita Lopez-Pedersen, Associate Professor, Department of Special Needs Education, University of Oslo
15.50	**Thank you for today**

Programme

Day 1: Thursday, 23 November 2023

Time	Room	
8.30-9.00		Registration & morning coffee
9.00-9.20		Welcome event
9.20-10.30	Fest-salen 2nd floor	Keynote 1 **Research on people with dyscalculia: What do we know and where should we go?** Professor Marie-Pascale Noël, Psychological Sciences Research Institute, University of Louvain (Belgium)
10.30-10.50		BREAK & WALK TO PAPER SESSION A
10.50 - 12.00		Paper Session A
	A401	**GROUP 1** **Student Jacob Frølund Davis, University of Southern Denmark (DK):** *Creating a Deeper Understanding of Dyscalculia with a simulation* **Speech Therapist Jonas Walfridsson, Danderyd University Hospital (SE):** *Specific Mathematical Difficulties – Diagnosis and Pedagogical and Didactic Actions*
	A403	**GROUP 2** **Associate Professor Anita Movik Simensen The Arctic University of Norway (NO):** *Learning Opportunities for Students Achieving low in Mathematics* **Docent Anette Bagger, Örebro University (SE):** Care for the SEM - Students Learning in Mathematics
	A405	**GROUP 3** **Associate Lecturers Lise Dausen & Pernille Ladegaard Pedersen, VIA University College (DK):** *How do Manipulatives Influence Students' Understanding of Fractions?* **Consultant Pernille Pind & Associate Lecturer Pernille Bødtker Sunde, VIA University College (DK):** *Difficult Numbers?*
12.00-13.00		LUNCH & EXHIBITION
13.00 - 14.00	Room	Paper Session B
	A401	**GROUP 4** **Associate Lecturer Heidie Clemens, VIA University College (DK):** *Guidance Conversations About Students with Maths Difficulties - Potentials and Challenges* **Associate Professor Edda Óskarsdóttir & Assistant Professor Ósk Dagsdóttir, University of Iceland (IS):** *Developing a Course on Mathematics for all: An Action Research Project*
	A403	**GROUP 5** **Project Researcher Sonja Solkunen, University of Turku (FI):** *Building the Finnish Digital Research Ecosystem for Assessment and Interventions of Dyscalculia* **Teacher Dorthe Riedel & Mathematics Consultant, Mette Thompson (DK):** *Fostering Numeracy Skills and Understanding Among Students in Helsingør Municipality*
	A405	**GROUP 6** **Associate Lecturers Per Nygaard Thomsen & Frede Krøjgaard, VIA University College (DK):** *Upper Secondary Pupils' Thinking and Actions Patterns in Mathematics*

Day 1: Thursday, 23 November 2023

Time	Room	
14.00-14.15		BREAK & WALK TO PAPER SESSION C
14.15-15.15	Room	Paper Session C
	A401	**GROUP 7** **Associate Lecturer Lóa Björk Jóelsdóttir and Associate Lecturer Pernille Bødtker Sunde, VIA University College (DK):** *Strategy use in Multi-Digit Artihmetic: Findings from low Performing Grade 6 Students* **Associate Lecturer Pernille Ladegaard Petersen, Professionshøjskolen VIA (DK):** *How students explain and understand different variables?*
	A403	**GROUP 8** **Teacher Bibbi S. Visby, Adult Education Center Fyn (DK):** *Care for adult learners' confidence at a danish adult learning center – a development study* **School Consultant Terje Engh Wiig, Oslo Education Agency (NO):** *Targeted and Organized Mathematics Teaching for Students who Perform Poorly in Mathematics*
15.15-15.30		BREAK & WALK TO PLENARY
15.30-16.15	Festsalen 2nd floor	Keynote 2 **Trends in Research on Spontaneous Focusing on Numerosity - Implications for Mathematics Education** Professor Minna Hannula-Sormunen, Department of Teacher Education, University of Turku (FI)
16.15-16.30	Festsalen 2nd floor	Summary of today's program
17.00-21.00		Conference dinner

Programme

Day 2: Friday, 24 November 2023

Time	Room	
8.30-9.00		Morning coffee
9.00-10.00	Festsalen 2nd floor	**Keynote 3** **Early Development of Number Knowledge: Identifying Risk of Learning Disability** Professor David C. Geary, Thomas Jefferson Fellow, Department of Psychological Sciences, University of Missouri (USA)
10.00 - 10.45	Festsalen 2nd floor	**Keynote 4** **Access to Mathematics - (How) Can We Create Equity and Inclusion in Mathematics Education?** Associate Professor Helena Roos, Malmø University (SE)
10.45-11.00		**BREAK & WALK TO PAPER SESSION D**
11.00-12.00		**Paper Session D**
	A403	**GROUP 9** **Associate Professor Lena Lindenskov, Aarhus University & Associate Lecturer, Bent Lindhardt, University College Absalon (DK):** *Dyscalculia test and teaching – results from three Danish Dyscalculia projects* **Associate Lecturer Michael Wahl Andersen, University College KP (DK):** *About teaching mathematics for students with general developmental disorders*
	A405	**GROUP 10** **PhD student Elin Nordbø, University of Stavanger (NO):** *A Meta-Analysis on the Relation Between Language Interventions for Students With Mathematical Difficulties in Primary School* **Associate Professor Cecilie Segerby, Kristianstad University & Associate Professor Christina Svensson, Malmø University:** *Implementing Linguistic Theories in Content-Included Mathematics Teaching*
	A200	**GROUP 11** **Docent Anette Bagger, Örebro University (SE):** *Proactively Pushing the Relational Frontiers of Mathematics Inclusivity: Enacting Disabled Student Driven Spaces of Rightful Presence*
12.00-13.00		**LUNCH & EXHIBITION**
13.00 - 13.45	Festsalen 2nd floor	**Keynote 5** **Effects of Mathematical Skills on Labour Market Outcomes and Life Conditions** Professor Helena Skyt Nielsen, Department of Economics and Business Economics, Aarhus University (DK)
13.45 - 14.30	Festsalen 2nd floor	**Panel Debate** **Nordic Representation** *Which Ways Are We Moving in the Nordic Countries - and what can we do together?*
14.30-15.00	Festsalen 2nd floor	**Closing of NORSMA & Announcement of the host of NORSMA 12**

Social events

DanSMa Reception
Date: 22 November
Time: 16.00-17.30
Place: DPU,
room A210 and A201

In connection with NCUM's Annual Conference /Teacher's Day on 22 November, DanSMa (Dansk Special Matematik) invites all participants to an informal reception.

The reception will provide excellent opportunities for networking with old and new colleagues, discussing lectures and workshops of the day, playing board games and having a good time. Drinks and light snacks will be served.

We hope you will join us!
www.dansma.dk

NORSMA 11 Conference dinner
Date: 23 November
Time: 17.00 - 21.00
Place: DPU

The conference dinner will start with drinks, snacks and mingling. At 18.00, a Nordic dinner buffet will be served followed by a traditional Danish dessert.

The dinner is included in the registration fee, but please note that you only have access if you have booked the dinner during the registration. If you are not attending, but have booked, please let us know.

We look forward to a nice evening with old and new colleagues!

01 - MEDELSTA 20 YEARS - A REVIEW

Arne Engström
Strömstad Academy

INTRODUCTION

At the second NORSMA conference in Örebro in 2003, a comprehensive account of the Medelsta study was presented (Engström & Magne, 2003; Engström, 2004). The study also attracted attention in the mass media. It is one of the larger studies of primary school pupils' mathematical skills in Sweden.

THE MIDDLE SCHOOL STUDY

The average study is a total survey of the mathematical skills of primary school pupils from year one to year nine in an average municipality over 25 years. The first study was conducted in 1977. It was then replicated twice, first in 1986 and then in 2002. During this 25-year period, three different curricula were in force: Lgr 69, Lgr 80 and Lpo 94. The same tests were used on all three occasions. This made it possible to study the outcome of mathematics education according to the three different curricula.

STATEMENT OF THE PROBLEM

A main hypothesis put forward by Magne (1958) was that students develop an increasingly lower level of performance compared to the specifications given in the curriculum for each grade. Other issues included how students' mathematical performance develops from grade to grade, what students achieve in different mathematical domains, and assessing the impact of different curricula on mathematical knowledge. Special attention was paid to the 15% lowest performing students.

RESULTS AT A GLANCE

Pupils' average knowledge increases steadily from grade to grade. There is a rapid increase in pupils' knowledge development mainly in grades four and five. In the final years, this trend levels off. There is no increase in the final years. With regard to the development of the performance of the new material acquired by the pupils during a year, it can be seen that retention decreases with each grade. The decline for the group as a whole is explained by the fact that pupils below the median, and especially the lowest performing pupils, have a lower retention rate with each passing year. For the 15% lowest performing pupils, the decline is very large. By the end of Year 9, their performance is on a par with the average pupil in Year 4. The curricula in force during the implementation of the Medelsta project have been very different from each other. One might have expected the students' performance to differ in some way between the three survey years. Instead, the outcome is the opposite. There are striking similarities in performance between the different surveys. The changes made to the curriculum have not had any impact on student performance.

20 YEARS LATER

It is now 20 years since the second conference in Örebro. Demographic changes through high levels of migration, the emergence of independent schools and free school choice, as well as changes in curricula and the Education Act's regulation of school support for low-achieving pupils, have changed the school landscape. Someone ought to investigate the consequences for the lowest performing students, sum up other empirical studies of student achievement that are relevant, and finally invite a comparison with developments in the other Nordic countries.

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02 - INTERACTIVE RESEARCH - A WAY TO ACHIEVE ACCURATE SCHOOL DEVELOPMENT INTERVENTIONS?

Karoline Holmgren
Umeå University, Department of Science and Mathematics Education, Sweden;

ABSTRACT
This article is based on a school development and research project that was carried out with the aim of increasing achievement in mathematics at several primary schools in a Swedish municipality. The main purpose of the article is to describe and contribute to a better understanding of the interactive research method and how it can be used for theory development in combination with school development. Interactive research is a practice-oriented and participatory method that involves including practice participants in the research process with the aim of contributing to both theory and method development.

The initiator of the research and school development project on which this article is based was the head of the municipality's primary schools: Statistics and screenings had for several years shown low achievement in mathematics. Action was needed - but what, where when and how? This time, unlike previous mathematics development projects, the head of the school wanted to implement measures based on scientific grounds and proven experience. National reports show that the role and responsibility of the principal is crucial for schools that demonstrate successful change and development work (Skolinspektionen, 2023) and in this study it was the principal who initiated the desire to combine school development with research. The Swedish Schools Inspectorate, which is the institution in Sweden that annually reviews that the school lives up to its important mission, has for several years identified that many school principals need to take a more active responsibility to provide the schools with the right conditions (see e.g. Skolinspektionen, 2023): Each principal needs to orient themselves on the schools' situation and difficulties as well as their causes, in order to be able to assist principals and schools with accurate efforts.
Interactive research fits well with the above development needs at the principal's level, as one of the purposes of participatory research is to develop an understanding of the context of which the practice is a part (Lindholm & Axelsson, 2021). The interactive research approach used in this research and development project proved to be a method that enabled common forums where school actors could meet together with researchers to analyze, discuss and develop. The results show that interactive research can be a well-functioning method to contribute to increased understanding and knowledge of school problems. By including a holistic approach where theory and local practice are seen as equally important aspects, more accurate school development efforts can be initiated.

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03 – A Meta-Analysis on the Relation Between Language Interventions for Students With Mathematical Difficulties in Primary School

Elin Nordbø[1], Anita Lopez-Pedersen, Vibeke Rønneberg
[1]*University of Stavanger*

Students' mathematical competence when they enter school has an impact on their mathematical outcomes throughout school (Duncan et al., 2007; Purpura & Ganley, 2014; Wei et al., 2013). Still, many students experience difficulties mastering the subject. Early identification and effort can reduce the difficulties before they get permanent (Fuchs, 2005), and it is, therefore, necessary to investigate how to help the students most properly. Language is a central part of students' learning process. Different models (e.g., "Triple Code Model" and "Pathway Model") include language as an essential part of learning early numeracy skills (Dehaene, 1992; LeFevre et al., 2010). Further, many studies have investigated how students' language competence impacts their mathematical outcomes (e.g., Fuchs et al., 2020; Hornburg et al., 2018; Purpura et al., 2017). Some (e.g., Fuchs et al., 2020; LeFevre et al., 2010) have focused on the student's general language competence, while others (e.g.,

Hornburg et al., 2018; Purpura & Ganley, 2014) have focused on the students' skills related to the mathematical concept (Powell et al., 2021). Although the studies have investigated distinct aspects of students' language skills, they all agree that weak language skills indicate an increased risk of developing mathematical difficulties. Furthermore, weak working memory is associated with poor mathematical skills. However, working memory tests require language skills (Melby-Lervåg et al., 2012), and therefore it may be weak language skills, not a weak working memory, that causes the difficulties. The question is whether explicit practice of language skills will improve the students' mathematical skills. Based on these reflections, I want to investigate what previous interventions dealing with enhancing language skills among students at risk for developing mathematical difficulties indicate. The investigation will be through a meta-analysis. The meta-analysis will provide an overview of the research field and give valuable information for further investigation. Included studies must be interventions dealing with mathematical difficulties, language, and students between 5 and 10 years old.

The preliminary research question is "What is the effect of language-related interventions aimed at students at risk of developing mathematical difficulties (age 5-10)? What characterizes these interventions?" With a background in the research question, the hope is to uncover which interventions about language and mathematical difficulties that exist, the characteristics and effect size of these interventions, and whether students at risk for developing mathematical difficulties would benefit from the explicit practice of language skills. The research platform Rayyan will organize, manage, and systemize the articles discovered through the search process. My supervisors Anita Lopez-Pedersen and Vibeke Rønneberg will be co-authors in the meta-analysis.

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04 - SPECIFIC MATHEMATICAL DIFFICULTIES – DIAGNOSIS AND PEDAGOGICAL AND DIDACTIC ACTIONS

Jonas Walfridsson[1], Ann-Louise Ljungblad[2], Helena Roos[2], & Ulf Träff[3]
Danderyds Sjukhus AB[1], Malmö Universitet[2], Linköpings universitet[3]

AIM AND RESEARCH QUESTIONS

This is a presentation of an interdisciplinary project to investigate how diagnosis of specific mathematical difficulties and pedagogical and didactic actions for students who meet the diagnostic criteria for the diagnosis of dyscalculia (World Health Organization, 2022) can be developed and linked together to result in a more equal and health-promoting life situation for this group of students. Dyscalculia is a diagnosis that refers to persistent difficulties in acquiring basic arithmetic and/or mathematical skills, such as number sense, memorization of number facts, accurate calculation, fluent calculation, and accurate mathematic reasoning. The difficulties should result in significant impairment in the individual's academic or occupational functioning (World Health Organization, 2022).

The aim with this upcoming project is to work with the diagnosis procedure regarding specific mathematical difficulties and didactic and pedagogical actions simultaneously on a national, regional, and municipal level for equal healthcare and education in Sweden.
This is investigated by two research questions:
How can an interdisciplinary collaboration between, on the one hand, institutions with expertise in the investigation of specific mathematical difficulties, and on the other hand institutions that design pedagogical and didactic actions be developed at a municipal, regional, and national level?

In what ways can pedagogical and didactic adaptations regarding specific mathematical difficulties be designed so that investigation and the designed adaptations can be linked together on both an individual, group and organizational level to favor fruitful participation in mathematics teaching for students in specific mathematical difficulties?

DATA AND METHOD

The project uses a mixed-method approach with a focus on design-based research (Bell, 2004) and single-case studies (Kazdin, 2011) and will collect data via interviews and interventions. The design-based method will be used to create a more thorough understanding of the development of a national system for diagnosis and pedagogical and didactical actions, whereas the single-subject design will be used to evaluate the effectiveness of different targeted interventions. One important mission of the project is to chart the situation within the Nordic countries with regards to how specific mathematical difficulties are discovered, diagnosed and treated in the societal system of each country, which might entail further collaboration within the NORSMA Network.

RELEVANCE

From an equity and health-promoting perspective, the project aims to develop interdisciplinary knowledge and competence regarding students in specific mathematical difficulties and their well-being, quality of life and educational opportunities and, by extension, also link this to their professional life and community participation. Hence, the project has a health-promoting potential that can make a big difference for many students studying mathematics within both today and tomorrow's school.

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05 - WHEN THE FOUNDATION FOR THE FIRST STAGE OF ALGEBRAIZATION IS MISSING

Pia Beck Tonnesen
University of Copenhagen and University College Copenhagen

INTRODUCTION

In secondary school, basic algebra is a crucial bridge between arithmetic and more advanced subjects involving functions and analytic geometry. However, it is a longstanding and widespread problem that large groups of students seem to get stuck at (or even before) this bridge (e.g.
Herscovics, & Linchevski, 1994). This has major personal and societal consequences because basic algebra as taught in lower secondary school plays a crucial role in students' further education.
School algebra is often described as a central gatekeeper (Loveless, 2013). To narrow the school algebra problem, we will investigate the transition from arithmetic to algebra and to gain knowledge about what algebraic techniques and theory are particularly problematic for students in Danish lower secondary school.

THEORETICAL FOUNDATION

The Anthropological Theory of Didactics (ATD) is used as the theoretical framework. A central feature in ATD is the use of praxeology to model school mathematics activity. A praxeology compromise types of task, techniques, technologies, and theories (Bosch, 2015). The "practical block" or praxis is formed by the types of task and by the techniques used to solve them (Barbé et al., 2005). The "theoretical block" or logos consists of technology and theory. In this case we define school algebra as a process of algebraization, a practical and theoretical tool to carry out modelling activity related to any school mathematical praxeology. To detect what kind of school algebra is problematic to lower secondary students we will use the three-stage model of algebraization (Ruiz- Munzón et al., 2013). In the three-stage model of algebraization, arithmetic can be identified as the domain of calculation programmes (CP). The first stage of algebraization is when you consider the CP as a whole and not only as a process. The second stage is introducing letters as parameters and unknowns, to model the relationship between CPs. The third, and last stage of the algebraization process appears, when the number of arguments of the CP is not limited and the distinction between unknowns and parameters is eliminated (Ruiz-Munzón et al., 2013). The three-stage model will be exemplified in the presentation and extended paper.

RESEARCH QUESTION

Based in the theoretical foundation we ask the research question "What arithmetical techniques and theory are problematic for students in Danish lower secondary school at the first level of algebraization? And "How does the students arithmetical praxeologies affect their further algebraization process?

METHODOLOGY

To answer the research questions, we will use a diagnostic test tool described in (Tonnesen, 2022). The test will diagnose what type of arithmetical tasks and corresponding techniques are problematic for the students and give insight into the level of theory.

RESULTS

The results are based on 69 Grade 7 students, and 22 Grade 8 students answers to the 45-minute unaided paper-and-pencil diagnostic test conducted in January and February 2022. The results show a large group of students struggle with simple arithmetical tasks, as they apply inefficient techniques to solve the tasks, which leads to further problems with the algebraization process.

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07 - PROACTIVELY PUSHING THE RELATIONAL FRONTIERS OF MATHEMATICS INCLUSIVITY: ENACTING DISABLED STUDENT DRIVEN SPACES OF RIGHTFUL PRESENCE

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Educational systems around the world are highly dependent on assessment. Nevertheless, the social justice contours of inclusivity are rarely assessed. In mathematics, assessment displays features that are quite similar around the world and across different levels of education: being repeatedly reported as essentially based on individual testing and depoliticized. Our developing research project challenge the grounds for this systemic practice and the perspectives that justify it, taking an alternative stance which re-thinks mathematics assessment through the Foucauldian lens in combination with the sociopolitical notion of rightful presence which science educators such as Calabrese Barton and Tan (2020) have developed and implemented in the United States. As such, we seek to offer tools for analyzing and re-defining the frontiers of mathematics assessment, particularly as it pertains to inclusive equity practices targeting sociopolitical enactments of justice driven by disabled students.

Core concepts in the re-thinking of assessment
Our approach to re-think mathematics assessment draws on Foucault's (1988) conceptualization of technologies of power to analyze the discourses and the relational unfolding of our interactive work as collaborative researchers. Through this dynamic perspective we can offer an example of critical, theory-based reflection upon the new frontiers of mathematics assessment. We model our relational reflection and collaborative subjectivization approach after Bagger and colleagues' (2018) work which delineates our approach in terms of methodology. The dialogical contours of this methodology are grounded on caring and sociopolitical principles of inclusion modeled after the tenets of rightful presence. We will initiate a systematic exploration of borders, epistemological power and relational rigor in conjunction to comparative illustrations of rightful presence in action across several countries. We aim to understand how they are reflected in the discourses of each of us as researchers and reflexive participants in this project. This will be carried out through us in our subjectivizing role as researchers having a discussion regarding inclusive dimensions of assessment in mathematics via e-mail. This will thereafter be collectively analyzed in regards to the core concepts of the project. In terms of outcomes, we expect to illumine epistemological dynamics of uncaring assessment application not merely as unintended contours of mathematics education but rather as sociopolitical mechanisms of exclusion. In turn, we aspire to develop concrete proposals for engendering disabled student-centered mechanisms of rightful presence in the classroom which could push the frontiers of caring inclusionary assessment beyond their current ideological disguises of objectivity and neutrality.

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08 - Targeted and organized mathematics teaching for students who perform poorly in mathematics

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PRESENTATION OF THE RESEARCH PROJECT
We would like to present the findings from a research project that was conducted in Oslo Schools over three years and concluded in February 2020. The initiative was conducted as an experiment, where 24 of 48 secondary schools in Oslo and 9 of 17 relevant upper secondary schools were randomly selected to participate (approximately 3000 students, annually 500 students in grade 8 and 500 students in VG1 over three years). The initiative consisted of two parts: Training of teachers in didactics adapted to students with low competence in mathematics and adapted teaching for students in a clearly defined target group for two periods of four to six weeks in grade 8 (lower secondary school) and VG1 (upper secondary school). A majority of the target group pupils received adapted education in small groups, consisting of pupils with weak results from national tests, the other target group pupils in larger groups. We found that pupils who received adapted teaching from trained teachers in grade 8 achieve a result on the national test in arithmetic in grade 9 that is about 6 percent of a standard deviation (equivalent to 0.6 scale points) higher than the comparison group, and perform less at the lowest levels of mastery, i.e., we find a significant positive effect on pupils' performance.

Some of the results summarized:
- Students who receive facilitated instruction in small groups perform about 0.6 scale points (6 percent of a standard deviation) better than the expected national test in arithmetic at grade 9.
- We do not yet know whether the effect will persist. Based on previous studies of the relationship between schools' contributions at secondary school and later results, the effect we find on NP9 corresponds to 0.6 percentage points higher completion of VGO and an effect on future income that in itself makes the measure socio-economically profitable.
- We do not find effects on the results of pupils who received facilitated teaching in large groups.
- In the first year, some pupils were taught in small groups by teachers who had not received training. We do not find effects on the results of these students.
- The previous two points suggest that the combination of training and small groups was necessary to achieve an effect. These results are less precise than the impact estimate for students with small group instruction. Teaching in large groups
large groups and group size in the first year are also not fully comparable with teaching and group size in small groups with facilitated teaching. We therefore have better evidence for the conclusion that the combination of training and small groups was effective than for the conclusion that each component was not effective on its own.
- Nor do we find any effects on other pupils, either positive or negative (positive effects could have been obtained if it was beneficial for the remaining pupils to be placed in small groups, and negative effects if, for example, redistribution of teachers led to poorer teaching).
- Responses to similar surveys from teachers in control schools suggest that the content of the training is not unique to this intervention. Comparing teaching practices in intervention and control schools is challenging, partly because we were only able to observe a limited number of schools, because not all teachers respond to surveys and because it is difficult to formulate questions that provide similar information from teachers who have and have not received training. Nevertheless, there appears to be a difference in the extent of teaching in line with the intervention, including more use of rich tasks and a higher proportion of students who are active in class. This difference in teaching practice appears to be due to the intervention and is probably the reason for the effect on results.

Didactic principles in the research project
Six principles for teacher training. The teacher is encouraged to:
1. Create coherence between sessions
2. Use low floor and high ceiling activities
3. Initiate MO(tivation) - (Per)FORMANCE
4. Conversation and initiating conversations that support understanding
5. Set realistic, high expectations
6. Use a journal

These principles are concretized in four methods and tools:
- "Thinking Blocks" - the block method - the Singapore model method
- Learning partner
- IGP: Individual, Group, Plenary
- Conversation moves - Math conversations that support students' understanding

Workshop: How can we include all students through exploratory work with "ordinary" math problems such as 18 * 5 = ?
Inclusion is a fundamental principle in the new curriculum - LK20. In the workshop, we show examples from the session plans in "PBG" on how teachers can realize the core elements of an inclusive community that stimulates exploration; a community where students are given

opportunity to think, discuss and reflect together, a community where students simultaneously practice democratic values and develop in-depth understanding through different representations. The teacher's role is to ask questions to find out what the pupils know or understand, listen to how they think/reason and challenge them further.

Representations: https://www.matematikksenteret.no/sites/default/files/attachments/Elever%20s om%20presterer%20lavt/P4_M1Representations-in-mathematics_fagtekst.pdf

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09 - Fostering Numeracy Skills and Understanding among Students in Helsingør Municipality

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The aim of this project was to support development of numeracy skills and understanding for all students, giving them an opportunity to learn (Hiebert, 2003). Children's number sense as the ability to decompose numbers (Sowder, 1992) or understanding of numerical magnitudes is closely related to their general math achievement (Laski & Siegler, 2007). Understanding in mathematics is making connections between representations as a part of an internal network. The degree of understanding is determined by the number of links and strength of these connections (Hiebert & Carpenter, 1992). Therefore it is important to foster numeracy skills and understanding for students in primary school.
In order for teachers to support this development, they need targeted knowledge about their students' skills and understanding. With this premise as a starting point we developed a test to measure numeracy skills among students in Helsingør Municipality, Denmark. The test was designed to assess key aspects of numeracy skills and understanding: subitizing, number sense, numerical magnitudes and symbol proficiency.
Based on a review of existing tests available in Denmark, and consulting with educators, we developed items in 10 levels with embedded progression in numeracy skills and understanding. From mastering and understanding natural numbers to mastering rational numbers. This test was pilot tested with a sample of students in a local school. The final tests consisted of 40 items in each level and were tested on 300-400 students. A Rasch analysis showed that the test can reliable and valid measure numeracy skills and understanding of students in Helsingør Municipality. The students test scores are used for criterion-referenced classification of proficiency levels.
Through our work and use of testing, we have become aware that students who have a low classification often have difficulty learning number names in the early grades, and this can impact their ability to learn math. It's a specific Danish challenge that the names of numbers are not linguistically logical in their structure, and therefore the symbolic proficiency is something that these students struggle with. But there is no common understanding of the scope of the problem with connection between knowing number names and development in mathematics.
Our developmental project contributes to the discussion on what we can do in schools if we dare to assess what we find as key components to learning math. The presentation will give a brief overview of a test to assess the numeracy skills and understanding developed in primary schools in Helsingør. A few examples form the basis for discussion of students' proficiency levels and what we have found that indicates a delayed understanding. The presentation will outline examples of interventions that teachers in Helsingør have initiated based on test results, with the aim to support students with delayed numeracy.

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10 - IMPLEMENT A LINGUISTIC THEORY FOR CONTENT-INCLUSIVE MATHEMATICS EDUCATION

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ABSTRACT
Several researchers highlight the importance of working with language in all subjects. However, research shows that the dominant way of working in mathematics lessons in Sweden is individually through math books (Boesen et al., 2014; Roos 2020). This approach in Sweden is unique, as the mathematics book is considered the main source of how students are offered mathematical concepts. A successful way to achieve a change in the current way of working is to start with the teachers' problem formulations about their teaching (OECD, 2019). The mathematics book itself is not problematic, but rather how teachers deal with the content to offer a content-inclusive mathematics education for all students. Previous studies have identified success factors such as collaborative conversations together with a supervisor, supported by scientific theories for changes in mathematics education, based on teachers' own problem formulations (Holmqvist, 2017; Segerby, 2017; Svensson, 2022). This has led to the following problem formulation for our study: How can scientific methods help to increase teachers' understanding of content-inclusive mathematics teaching based on linguistic perspectives?
The study involves six teachers in mathematics, one school leader, and two researchers, and students in grades 2-6 participate. Initially, the teachers had identified different aspects of the position system as problematic for students. The methodological approach is the Learning Study based on the scientific theory of scientific theory of Systemic Functional Linguistics (Halliday & Hasan, 1985). Systemic Functional Linguistics (SFL), which is a socio-linguistic theory based on the assumption that every text is about something (the ideational metafunction), is addressed to someone (the interpersonal metafunction), and is based on text structure (textual metafunction) (Halliday & Hasan, 1985). The ideational metafunction refers to what is happening, that is, what it is that the participants are engaged in, where the choice of concepts relevant to the context is in focus (Halliday & Matthiessen, 2004). The interpersonal metafunction is based on who participates and what roles and status they have as well as the type of status they have and the type of relationships that arise between the participants (Halliday & Hasan, 1985). The textual metafunction is based on the role that language plays and what the participants expect the language to do for them in each situation (Halliday & Hasan, 1985). The textual metafunction is based on how these different representations work together to make the text coherent.
We, teachers and researchers, have together constructed pre-and post-tests, planned, implemented, and evaluated how students have discerned the mathematical content. All research lessons and collaborative conversations have been video recorded. The theoretical tool SFL has been used both for the planning of the lesson and in the subsequent analysis of the lesson, which has correlated together with the results of the pre-and post-tests conducted. The collaborative conversations have then been studied to discern how the systematic learning study methodology has enabled a better understanding of content-inclusive education for all students.
What is noteworthy in this study is that based on the teachers' own identified problem statements, SFL (Halliday & Hasan, 1985) has been a significant factor in increasing teachers' understanding of content inclusion from a multilingual perspective. During our presentation, we will explain more about the results.

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11 - CREATING A DEEPER UNDERSTANDING OF DYSCALCULIA WITH A SIMULATOR

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BACKGROUND
For a non-dyscalculic person it can be difficult to comprehend the difficulties dyscalculic persons experience. This is also the case for mathematics teachers. If you cannot comprehend the difficulties, it can be difficult to spot and act on them as well. With this developmental project we wanted to support a better understanding of the difficulties dyscalculic persons experience, especially in the mathematics lessons. We seek to answer the following questions:
Can a simulator experience provide a deeper insight into how a dyscalculic person experience numbers and the processing of numbers?
Can this insight support mathematics teachers in planning and organising classroom teaching to better support dyscalculic students?

DEVELOPING THE SIMULATOR
The design of the simulator was guided by research and interviews with expert researchers and practitioners as well as a person with dyscalculia. We developed three series of tasks typical of a mathematics lesson and manipulated them to simulate the difficulties a dyscalculic person would experience. One example is a calculation task written in blue but disguised with red text and drawings making it difficult to find the numerical information necessary to perform the calculations. Furthermore, the calculator on which the participants were to do all calculations were manipulated during the participants work; numbers were randomly assigned and occasionally a number was missing.

Testing the prototype
The first prototype was tested on three university students without mathematical difficulties. Each participant completed a 25 minute session in a setting simulating a classroom lesson. The participant's actions and responses were videorecorded for further analysis.
To evaluate the use of the simulation with teachers, we asked teacher students to describe what behaviour they noticed in the video recordings and reflect on how students with dyscalculia or severe mathematical difficulties could be supported in the classroom.

Preliminary results
From the video recordings, we can observe a range of behaviour typical for students in mathematical difficulties, such as frustration, loss of trust in own ability, resignation and indifference towards the tasks. From the work with the teacher students we found that the video recordings provided them with an insight into what a dyscalculic student endures during a normal mathematics lesson. They engaged vividly in discussions on what aspects of the tasks and "the teacher's" behaviour that enforced or remedied the participants experienced difficulties. This led to suggestions on how to support students in the classroom. From these preliminary results, it seems that a dyscalculia simulator has the potential to provide insight into how students with dyscalculia experience numbers and a typical mathematics lesson.

NEXT STEP AND PERSPECTIVES
The next step is to develop an online platform where the simulator will be made available together with background information on dyscalculia. This will be presented at the conference. We hope to be able to give teachers and future teachers a better insight into how a person with dyscalculia can experience numbers and mathematics, and thereby be able to give them the help they need.

12 - Care for the SEM-students learning in mathematics

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Dilemmas of diversity and the balancing act between protection and participation in learning as reported by Norwich (2014) is present at a high degree in teachers' education of students in need of special education in mathematics, SEM-students (Bagger & Roos, 2015; Bagger et al., 2019; Roos et al., 2020). This concept resonates with international research in the field of inclusive education, relational pedagogy and Magnes approach to SEM-students (2006). A well reported dilemma in the teaching of SEM-student is the dichotomy between challenging and promoting participation in learning and the protection of SEM-students' opportunities to positive identity building as knowers in mathematics to count on (Jensen & Bartell, 2013; Long, 2011; Nicol et al., 2010). How these two can meet in an efficient way and in which care for the learning of the SEM-student includes both protection and participation (see Watson, 2021) is left un-answered. This paper reports on the preliminary findings in a study on the care for the SEM students learning in mathematics following three research questions: RQ1) How does care for mathematics manifest in research? RQ2) How does care for the learning manifest in research? And finally: RQ 3) How do these two intersect, in the care for the learning in mathematics for the SEM-student, in research.

Methodology
We draw on Watsons (2021) model of care for the learning in mathematics, to select and analyse research on the education of SEM-students (students who are challenged or face obstacles in their learning, who need something else than what is offered in the everyday classroom). We applied four core concepts to the systematic review: Care for the learning, Care for mathematics and Care for the learning in mathematics and SEM- student. We searched for math* and care* in the database ERIC1. We de-selected texts no related to SEM-students and performed a thematic qualitative content analysis and searched for themes and patterns within and between the RQs (Braun & Clarke, 2006).

Results and conclusions
Theoretically, Care for the mathematics and Care for the learning and their intersection represents an account of the Care for the learning in mathematics for SEM-students. Preliminary findings show that care for the mathematics and the learning indeed intersect and is hard to clearly separate in presentation of results. RQ1: Care for the mathematics in research often resonated with student's opportunities to participate in learning and regarding specific areas as problem solving, counting, geometry and teaching metacognitive and cognitive strategies as well as mathematical systems and operations (for example Hackenberg, 2010). RQ 2: Care for the learning concerned how to best understand students understanding, challenge their thinking and support mathematical development.
Key in this was also high expectations (Maloney & Matthews, 2020; Ransom, 2020; Ellerbrock & Vomvoridi-Ivanovic, 2022; Hunter & Stinson, 2019). RQ3: In sum, Care for the SEM-students learning in mathematics, in terms of protection and participation (see Norwich 2014), circle around ethics, culturally sensitive teaching and awareness of the community and students' prerequisites (Ellerbrock & Vomvoridi-Ivanovic, 2022; Maloney & Matthews, 2020; Bartell, 2020).

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13 - Developing a Course on Mathematics for all: An Action Research Project

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Mathematics for all is a graduate level course taught at School of Education, University of Iceland. The course focuses on theories and research on how children learn mathematics and how teachers can design and adapt curriculum as they teach mathematics to diverse groups of learners. Participants learn about various teaching strategies and resources as they study teaching and learning in inclusive education settings.
The purpose of this action research is to gain an understanding of how we can develop our graduate level course Mathematics for all. The aim is to gather data from course participants on how the course has impacted their professionalism and mathematics teaching. The research question is: How has the course Mathematics for all impacted the professionalism and mathematics teaching of participants?
The research method is educational action research based on the action research spiral and is organised in three cycles:
First cycle is based on data collection from the fall of 2022 and data consists of course material, participants' assignments and their qualitative evaluations of the course;
Second cycle is based on data collected in the fall of 2023 that consists of interviews with participants a year after they attended the course as well as classroom observations;
Third cycle is focused on reflecting on and analysing all the data as well as implementing changes in our course on mathematics for all that will be next taught in the fall of 2024.
Throughout the research cycles, we write research journals that are treated as part of the data. Data will be analysed thematically using course themes as basic categories. 12 of 20 student teachers taking the course in fall 2022 agreed to participate in the study. The group represents all school levels.
Findings from the first phase indicate that the participants found the course helpful for developing their own teaching. They expressed that the readings and tasks presented in the course had enriched their understanding of themselves as mathematics learners and doers. They described the value of learning about Universal Design for Learning, creativity in mathematics and different ways to support mathematics learning for all learners. Those working as teachers (about half the group) while participating in the course explained that they had employed their learnings from the course in their classrooms, including mathematical discussions, formative assessment and open problem solving. Those not currently teaching concluded that they would use their learnings from the course in their future teaching.
The planned follow-up interviews and observations in fall 2023 will provide a more thorough insight into how sustainable the learnings from the course have been for the participants and also where we can develop the course further.

14 - How do manipulatives influence students' understanding of fractions?

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The use of various concrete fraction manipulates have been suggested as a way to support students' fraction knowledge development, but few studies have investigated how these concrete manipulatives can both support and distract students' mathematical reasoning. This led us to the following overarching question: How are students with mathematics difficulties supported or distracted in their reasoning by the use of concrete manipulatives compared to high performing students when comparing fractions?

Procedure
This study is part of the TRACK research project (Teaching Routines and Content Knowledge), which aims to develop key elements from a Singapore-based approach into the Danish school context. The project started in 2018 and will continue until 2027. The participants are middle school students.
The key elements are problem solving and the consistent use of different representations (i.e., concrete, pictorial and abstract), including a special emphasis on the use of manipulatives. The first school year the intervention took place, we conducted semi-structured interviews with ten fourth grade students during which the students were asked to explain how they would compare different fractions. For this study, we focused on two students, who were chosen, based on their low scores in a curriculum-based assessment targeting fraction proficiency and on their teachers' evaluation, defined as having mathematical difficulties.

Result
The students having mathematical difficulties showed a tendency to whole number bias when they were asked to compare the magnitude of two fractions. The bias means, that when the students were given the fractions 1/4 and 1/5 to compare, they would identify 1/5 as bigger as they related to their knowledge about natural numbers where five is bigger than four. They were then asked to explain why 1/5 was bigger. During the interview, their reasonings were based on the manipulatives and showed that their misconception about the size of the symbol representations was contradicted by their reasoning while using the manipulatives.
Our preliminary result showed that the manipulatives supported them to overcome their whole number bias connected to the representation aspect, meaning that they developed an understanding of the fact that the numerator and denominator were a part of a whole relation and not separate numbers. At the same time, the students showed that the solving process stayed at the same level of abstraction during the interview and very little progress were found. In other words, their reasoning was connected to each task and, when they were asked to compare two new fractions, they did not use their experience from previous tasks.

15 - Relationships between student's affective and mathematical development and the classroom culture

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In Denmark, students change schools when they transition from primary school to upper secondary school, and some students experience changed behavior and attitudes towards school and subjects as a result of this change.
The aim of this project is to investigate the positive change that occurs for students who previously have struggled with lack of motivation and learning in mathematics, why this change occurs and when it occurs. We see the student's affective development as a continuum from math anxiety to math confidence.
Pupils' experience of a safe learning environment and good relationships is of great importance for their learning. However, this field is not particularly well documented when it comes to students with challenges in mathematics at Danish upper secondary level.
In our analysis, we mainly use Paul Cobb's theory to describe the classroom culture.
We want to explore the above problem area based on the following questions:
How is the connection and mutual influence between the student's affective development, the student's mathematical development and the development of the classroom culture?
How is this reflected in the development of students' thinking and acting in the classroom?
The study includes 8 students who follow C-level mathematics in two different classes (C-level is basic high school level). All 8 have been identified both by screening and through interviews at the start of the school year as having challenges in mathematics. This is also confirmed by the students' own statements.
All 8 participants are interviewed twice.
In January, the students were interviewed in pairs on the basis of group work.
In April, individual timeline interviews are carried out, where the students' experience of the development of their attitude to mathematics, their mathematical development and the development of the classroom culture are examined with a focus on identifying significant elements, e.g. 'critical incidents'.
The preliminary studies of group interviews show that the majority of those interviewed have experienced a positive change in confidence in their own abilities, pointing at the safe learning environment and the teacher relationship as important parameters in this connection. Furthermore, it turns out that it contributes to the feeling of security when the student is among students at the same academic level.
The group interviews showed that we needed to focus more on each individual's timewise development and therefore we needed individual timeline interviews in order for us to answer the research questions. The results of these will be presented subsequently. The group interviews will mainly be seen as background material for the individual interviews.
The results of the study will provide knowledge about how upper secondary pupils experience the mutual connection between their affective development, their mathematical development and the classroom culture and how this is expressed in their thinking and action patterns. Overall, it will contribute to an increased understanding of the processes that students go through in connection with positive changes.

16 - Difficult numbers?

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Introduction
Some numbers appear to be more difficult than others. It will come as no surprise to teachers, that children find single-digit multiplication with operands 7 and 8 more difficult than multiplications with other operands (Taraghi et al., 2014; van der Ven et al., 2015). These differences in difficulty levels have been explained by several aspects (see van der Ven et al., 2015), for example problem-size effect. However, existing research has primarily analysed difficulty through correctness of performed operations. How the difficulty of numbers is perceived irrespective of the individuals' ability to perform a calculation correctly, has not been systematically investigated. We present here a first exploratory study investigating the research questions: Are some numbers perceived as more difficult than others? If so, is this dependent on self-evaluated mathematical ability?

This study
In a survey conducted on Facebook, adults (age>18) were invited to answer a short questionnaire on number preferences and motivation to perform multiplications and calculations in context. Participants (691 adults: 515 women, 174 men) were not asked to perform the calculations, only to state on a scale from 1 to 5 if they believed they would be successful and if they would prefer to do the calculation or not. Questions on age, gender, and mathematical ability (self-evaluation on a scale from 1 to 5) and an open question on experiences with numbers in general was also included.
For the analysis the difficulty of numbers was scored from 1 to 6 based on the presence of digits following the findings of Taraghi et al. (2014) of difficulty of operands in single-digit multiplication: the easiest operands (e) were 1, 2, 5 and 10, intermediate difficult (i) were 3, 4, 6, 9, and most difficult (d) were 7 and 8. Difficulty level 1: ee or eee, 2: ei or eei, 3: ed or eed, 4: ii or eii, 5: id or eid, and 6: dd or edd.

Results and Implications
Overall, men reported all numbers as easier than women even after correcting for self-evaluated mathematics ability level and difficulty of numbers, and more interesting, low ability groups perceived difficult numbers (numbers including digits 7 and 8) as more difficult than high ability groups did. The differences in scores indicate that low achievers perceived difficult numbers as relatively more difficult than high achievers did. More detailed analysis will be presented and discussed at the conference.
Whether or not numbers are perceived as difficult by some achievement groups has implications for teaching as well as for research. If a person perceives a task as difficult from the presence of specific digits, that could influence performance and ability to engage in for example learning activities. As one participant stated: "I am a math teacher, but even so it feels like a shock through my body if I have to do arithmetic with 7 and 8 and the result exceeds 10." These aspects of individual differences in perceived difficulty are relevant in teaching as well as research.

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17 - The relationship between affective factors and basic numeracy skills in pre-service teachers

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Supporting and promoting students' learning is one of the most essential responsibilities of a teacher and it is well established that teachers play a pivotal role in students learning in mathematics. Previous studies have found that teachers' content knowledge (i.e., basic numeracy skills) and affective factors influence students' numeracy skills, self-concept, interest and anxiety. Especially in mathematics, teachers' content knowledge has a positive effect on students' performance (Bolyard & Moyer Packenham, 2008). Students to teachers who have good content knowledge and self-concept in mathematics are more likely to experience less mathematics anxiety and learn more in mathematics (Lau et al., 2022; Schaeffer et al., 2021). Contrastingly, teachers' math anxiety is negatively related to student's anxiety and performance (Schaeffer et al., 2021). The aim of this study was to investigate how affective factors (i.e., self-concept, enjoyment, utility value, and anxiety) are related to basic numeracy skills (i.e., number processing and arithmetic fluency) in pre-service teachers.
The sample consisted of 111 teacher students (101 female; 7 male; 3 not reported) from three universities. The affective factors (i.e., self-concept, enjoyment, utility value, and anxiety) were measured with a twelve-item self-reported questionnaire, modified from the Fennema-Sherman Mathematics Attitude Scale (Fennema & Sherman, 1976). The questionnaire included both positive and negative statements regarding their affects towards mathematics (i.e., "Doing calculations in the head makes me feel anxious." or "Math skills and knowledge are important in everyday life.").
Students responded to the statement on a four-point Likert scale from strongly disagree (0) to strongly agree (3). Arithmetic fluency and number processing were measured with the digital Functional Numeracy Assessment – Dyscalculia Battery (FUNA-DB) (Räsänen et al., 2021). FUNA-DB consist of six tasks: number comparison, digit dot matching, number series, single-digit addition, single-digit subtraction, and multi-digit addition and subtraction. The students participated anonymously and no personal information other than their gender and age was collected.

In the preliminary analysis, correlations analysis was used to establish the relationship between the affective factors, number processing and arithmetic fluency. Structural validity of the affective measure indicated that a four-factor solution fitted the data best, $\chi^2(48) = 75.3$, $p = .007$; CFI = .960; TLI = .945; RMSEA = .072. The four factors were identified as self-concept, enjoyment, utility value, and anxiety. Coefficient alpha estimates for the total scale were .891 (self-concept, α = .848; enjoyment, $\alpha = .820$; utility value, $\alpha = .765$; anxiety $\alpha = .773$).

The preliminary results indicate that significant moderate correlations exist between self-concept (r = .430, $p < .001$), enjoyment ($r = .351$, $p < .001$), utility value ($r = .221$, $p < .05$), anxiety ($r = -.375$, $p <$

.001), and arithmetic fluency. However, none of the affective factors was significantly correlated to number processing. The preliminary analyses were conducted with the number comparison task and the calculation fluency task. At the NORSMA11 Conference, we will present results based on the complete FUNA-DB.

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18 - CARE FOR ADULT LEARNERS' CONFIDENCE AT A DANISH ADULT LEARNING CENTER – A DEVELOPMENT STUDY

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The presentation focuses on how care for self-confidence and mathematics-self-confidence is practiced by teachers at an adult learning center in Denmark in relation to adult learners that are heavily mathematically challenged and may be seen as challenged by dyscalculia.

THE CONTEXT

At the Adult Learning Center at the Danish island Fyn [In Danish: Voksenuddannelsescenteret VUC Fyn] mathematics classes take place as well at the lower secondary level as second chance [In Danish: på grundskolens sidste trin 8., 9. ,10. klasse] and at higher secondary level [In Danish: gymnasie/HF-niveau].
It is a characteristic for Danmark – contrary to for example Sweden – that curricula and national examinations for adults at lower secondary level are adapted to adults and differ from lower secondary for children, despite both lead to the same level of mathematics.
Also, mathematics curricula for adults at lower secondary level exist in two different forms. One is called General Adult Education in Mathematics [In Danish: AVU-matematik = Almen VoksenUddannelse i matematik]. See further at Fag og læreplaner | Børne– og Undervisningsministeriet (uvm.dk).
The other form is called Preparatory Adult Mathematics = PAE Mathematics [In Danish: FVU-matematik = Forberedende VoksenUndervisning i matematik]. See further at Forberedende voksenundervisning (FVU) | Børne– og Undervisningsministeriet (uvm.dk) and at Identitet, læreplaner og vejledninger | emu danmarks læringsportal
Too, PAE-mathematics is discussed in Lindenskov (2018).

DEVELOPMENT WORK

At the adult learning center, teachers have continuosly been involved in developmental work on adult learners' mathematics learning. Specific focus on the adult learners mathematics self-confidence has been prioritised.
Also, specific interest on the adult learners that are most challenged in their mathematics learning has been prioritised.
These mathematics teachers have been engaged in further teacher training, called Pedagogigal Diploma in adult mathematics.
At the adult learning center testing for a dyscalculia diagnosis is done by material including materials from the Swedish neuropsychologist Bjørn Adler. A positive test gives permission for the school to apply for financial support for extra learning sessions to the learners, in Danish called SPS.
Teachers' personal experiences from the development work includes that the focus on everyday life mathematics and the focus on functional mathematics which are central in PAE-mathematics have a positive impact on adult learners diagnosed with dyscalculia.
Anyhow, it is also obvious for the teachers, that their diagnosed learners underline from their experiences from their PAE-mathematics courses that all students in a PAE-class were expected to learn at the same pace, which is not more than a phantasy!
All diagnosed learners report that time is a big issue. They feel strongly, they do not learn at the same pace as the majority in a PAE-class. Also, some diagnosed learners report, their former mathematics teachers – including FVU-mathematics teachers – prioritized rote learning and were not able 'to hear' the learners' needs for understanding.
All diagnosed learners also report that their lacking mathematics confidence has a big impact on their life – not only formally in relation to enter further education and work they wish to, but also in everyday activities. The specific learners' everyday functional mathematics needs can be met in the PAE-mathematics, but it is a necessity that the class climate allows learners to communicate their everyday mathematics functional needs. It may for instance be, how to qualify to get and understand information about time schedules and

about time-length for changing from one train to another train.
Damaged mathematics- self-confidence therefore can certainly also damage general self-confidence. Teacher ought therefore in their practice to care for the learners' mathematics and general self-confidence just as much as they care for their mathematics learning. The question arises on how care for learners' self-confidence manifest in teachers' practice.
The experienced teachers report that it seems a necessity that the teachers support the learners in remembering and reflecting their mathematics learning through time. It seems the learners most often think that they learn nothing and do not progress. Then the teachers are the only ones who can confront this thinking: the teachers ought to help to document progress, smaller progress as well as bigger. For instance the teacher can tell the learner: Earlier you had no strategies for choosing, counting and calculating when you would buy stuff in the super market, but now you just told me that you start buying a number of chicken breast instead of always a whole chicken, and you told your family was shocked to see your development …….. etc. And Earlier, you did not know thousand, but now you do and yesterday you red and wrote the symbols of a million ……. etc.
Anyhow, there is a risk if the teacher takes over responsibility for remembering progress, The risk is, the learner cannot do without the teachers. This is a risk that must be openly communicated between teacher and learners. It must be communicated to the learners that in a moment they have to do without the teacher. This is also a result from the development study.
Finally, it is an experience from the development study that part of the learners diagnosed with dyscalculia also suffered other diagnoses.
Other parts of the development study, relevant mentioning are as follows:
Methodology for continuously observe and theorize learning activities, followed by testing the theory.
Organization of knowledge sharing among teachers.
Organization of knowledge sharing with other institutions.
Building up a number and activity park in order to draw mathematics out of class rooms.

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19 - GUIDANCE PROGRAMMES FOR STUDENTS WITH MATHS DIFFICULTIES - POTENTIAL AND CHALLENGES

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BACKGROUND

Studies by De Simone and Parmar (2006) and Clemens (2016) show that maths teachers need to have resource persons at their disposal when working towards academic inclusion. In Danish schools, maths counsellors are included as such resource persons.

NORSMA 11 presents results from a development project under R&D, VIA University College, on counselling conversations between mathematics counsellors and subject colleagues, where the counselling is based on issues from the mathematics teacher's teaching of students with mathematics difficulties.

The project draws its empirical data from a mandatory competence development programme for all schools in Aarhus Municipality aimed at second-year maths teachers, maths supervisors and educational leaders. The programme is called 'Early Intervention in Mathematics Aarhus' (TIMA) and is based on evidence-based research from 'Matematikindsats 2017' (Harder et al. 2020). The programme lasts one semester and has been run four times in the period 2021-23 with 12 participating schools per semester. 230 maths teachers, including 48 maths tutors, have participated in TIMA.

TIMA includes a sub-course for the maths supervisors, which consists of: 1) participation in a course on systemic coaching of 18 lessons, 2) holding internal systematic guidance courses for their own colleagues in TIMA and 3) participation in a systematic guidance course with an external supervisor from VIA University College.

In the internal counselling sessions, the intention is for mathematics teachers to present complex learning issues from their own teaching of students with mathematics difficulties. The aim is for the conversations to be practiced as systemic coaching (Moltke & Molly, 2019) with the aim of moving the guidance away from a counselling-dominated practice based on the EVA report (EVA, 2009), which states that resource persons can offer counselling when it comes to providing answers to teachers' specific questions, whereas guidance is necessary when it comes to complex issues. In the counselling process, the counsellor uses a framework of questions that can support the counsellor in establishing processes in which the teacher is given the opportunity to develop and expand their narratives about their own students in mathematics difficulties; their understandings of why students are in mathematics difficulties; their ideas about how the situation could be different; and their didactic action competence.

THE PROJECT'S RESEARCH QUESTION

What are the potentials and challenges of counselling between mathematics tutors and subject colleagues as a structured conversation practiced as systemic coaching based on video clips from colleagues' teaching?

DATA

Data consists of video recordings of the counsellors' supervision of their own subject colleagues, written notes from the counsellors' external supervision, surveys from the counsellors' teaching and a focus group interview with a team of counsellors. Data is analysed based on the research question.

RESULTS

The results show that the counsellors experience that guidance, as opposed to counselling, is a new task, and that there is a potential in conducting guidance as structured conversations practiced as systemic
coaching when working with students with maths difficulties. This form provides the opportunity and space for the counsellor and teacher to jointly uncover the teacher's challenges in situations with students with maths difficulties before decisions are made about which didactic measures the teacher should take in teaching the students and why. The results show that counsellors

need to practice structuring the conversation and develop their conversation techniques. The presentation of the results elaborates on the potentials and challenges of the structure and question framework used in the guide.

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20 - Building the Finnish digital research ecosystem for assessment and interventions of dyscalculia

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PURPOSE
The purpose for the initiatives presented at NORSMA11 is to combine and co-develop existing tools and systems in Finland into a research-inspired tool for co-creating the Finnish educational system and psycho-educational research.

FIRST INITIATIVE
In a multi-university research group, we have created a series of initiatives to narrow the gap between research and practice of dyscalculia treatment. The first of these initiatives was to build a family of research-based assessment tools for dyscalculia. This initiative runs under a project called FUNA (Functional Numeracy Assessment, Räsänen et al., 2021). Our database consists of over 55 000 FUNA-DB online dyscalculia screening assessments, with about 10 000 new assessments conducted during Spring 2023. FUNA automatically compares the student performance against a large and representative comparison group collected in the same educational culture. The teachers can change the comparison group freely, and the system offers updated results based on the teacher's choices. However, because both Finnish and EU legislations have been very resistant towards automatic decision-making and profiling, especially when concerning children and health or diagnostics, our system does not provide any automatic way to define any person who responds to the tasks as dyscalculic (ICD-10 F81.3 diagnosis).
In the forthcoming articles, we discuss the differentiation between low achievement and learning difficulties using FUNA-DB indexes (Hellstrand et al., submitted) and test-retest reliability (Hakkarainen et al., in preparation).

SECOND INITIATIVE
The second initiative is BOSS (Briefcases of Special Support) materials for special education research and practice. BOSS materials are digital training packages that are easy to distribute to individuals or groups and monitor each participant's progress. There are over 40 maths briefcases for special education, each meant for a weekly support session for 3 months. New materials are constantly under production. During the first year, we had collectively almost 2 000 unique users, i.e. children and youth using our support materials. The same BOSS distribution system will used in controlled intervention studies.

THIRD INITIATIVE
The third initiative, the TRILA-teacher network, was introduced in September 2022. All teachers or rehabilitation workers in Finland can join this network. By registering, TRILA-teacher will get free access to BOSS materials and, in the future, also to selected FUNA assessment tools. Free access is given for two years, which can be renewed by participating in one TRILA-consortium study. Over 500 teachers have joined this research collaboration network in just six months.
TRILA-consortium is an invitation-based international research network. The consortium members produce together digital assessment and learning tools. The membership gives rights to use any digital tools developed by the consortium in the researcher's studies on a cost-plus basis to cover only the administrative and technical costs. The consortium aims to offer researchers the possibility to develop digital assessment tools and rehabilitation materials and conduct diagnostic and intervention studies without the need to develop, maintain or administer a digital platform.

A COMMON DIGITAL LEARNING PLATFORM
Turku Research Institute for Learning Analytics (TRILA) maintains the digital technology needed to run these initiatives. TRILA is a multidisciplinary research unit at the University of Turku. The research laboratory aims to improve the quality of teaching and promote digital education using learning analytics in Finland and beyond. The institute has developed the UNESCO (2020) and UNICEF (2022) -awarded digital learning platform called ViLLE (Visual Learning

Environment, Eduten in international contexts). These initiatives described above are vital for transforming the platform into a research-inspired tool for co-creating the Finnish educational system and psycho-educational research.

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21 - DYSCALCULIA TEST AND TEACHING – RESULTS FROM THREE DANISH PROJECTS

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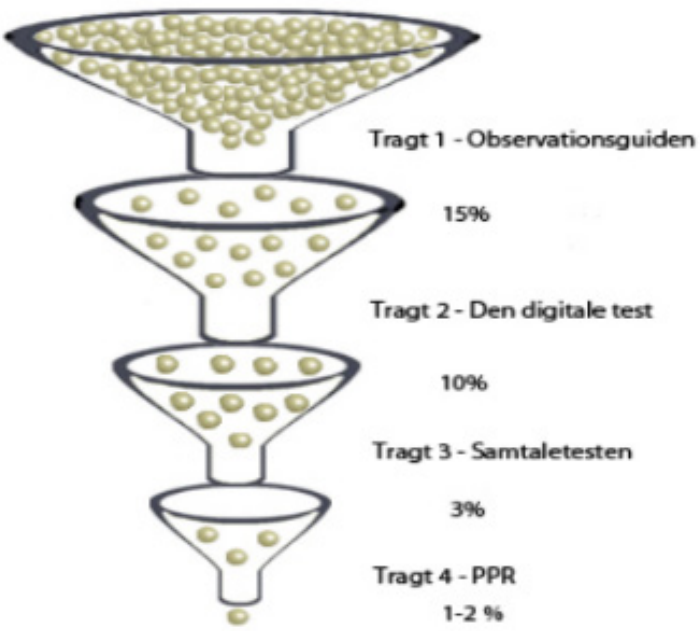
BACKGROUND
Discussions on dyscalculia have been on the agenda for several years in Denmark. Just after the NORSMA conference in Copenhagen 2013, the first signs for Ministerial recognition showed up, as the Danish Ministry of Education decided to initiate a development process towards a national dyscalculia test for school children grad 4 and guidelines for follow-up teaching. The political intention was to keep developmental, and implementation work as public affairs, financed through tax funds, instead of letting individual citizens and families pay for private dyscalculia consultancy.

TEST MODEL, ITEMS, AND TEACHING GUIDELINES – PROJECT 2014-2018
The project 2014-2018 was initiated and financed by the Ministry of Education. The methodology used was design-based research with literature surveys, interviews with self-declared dyscalculic adults, and cycles of empirical testing of collected and new-created test items and observation guidelines. The empirical testing took please among 4th graders around Denmark. Besides exemplifies teacher guiding material, the project resulted in definition and description of dyscalculia, a funnel test model with four elements, and test and observation items:



Report is published at
https://emu.dk/sites/default/files/2019-09/Talblindhedsprojektet_endelig%20april%202019.pdf

TEST MODEL, ITEMS, AND SOME INTERVENTION TRY-OUTS – PROJECT 2020-2021 TEACHING GUIDELINE – PROJECT 2020-2021
The project 2020-2021 was initiated and financed by the Ministry of Education for validating and improving results of the 2014-2018 project. The 2020-21 project developed the IT-design of the assessment, validated part of it, and tried out some interventions. April 2023 the report from 2021-2021-project was released by the Ministry of Education at

Rapport viser lovende resultater for en test, der kan finde frem til elever med tegn på talblindhed | Børne– og Undervisningsministeriet (uvm.dk)

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Based on the two projects, a following phase, 2 years long, is sketched, concentrating on experiments with hypotheses on what might support students which are hit by dyscalculia.

IMPROVING TEACHERS' AND MUNICIPALITY'S DYSCALCULIA KNOWLEDGE AND EXPERIENCE – THE TALRO-PROJECT

A two-year old development project was carried out in a major Danish municipality. The main aim of the project was to increase the municipality's mathematics tutors' experience with and knowledge of the phenomenon of dyscalculia, as well as investigate possible measures to improve the learning situation for dyscalculic pupils. The project included five mathematics tutors and five presumed students with dyscalculia from 4th to 6th grade in five different schools. The students were designated as dyscalculic by the tools developed in the projects 2014-2018 and 2020-2021, and by psychological test for general developmental disorder.

The methodology was design-based development with several cycles of observations, creating and collecting teaching activities, trying out the activities, and reflections by mathematics tutors. The mathematical content of activities was based on the previous investigation of students' mathematical difficulties in numbers, numerosity, and arithmetic. The activities were discussed and reflected at whole-day meetings among project manager and mathematics tutors and adapted to each individual student. All teachers wrote diary notes from all teaching sessions, and all teachers and students were pre- and post-interviewed. The results of the project show approaches and teaching methods that seems to benefit students with dyscalculia to different degrees.

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22 - STRATEGY USE IN MULTI-DIGIT ARITHMETIC: FINDINGS FROM LOW PERFORMING GRADE 6 STUDENTS

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THE STUDY
Aiming for strategy flexibility is an important goal in arithmetic (e.g. Baroody, 2003; Verschaffel et al., 2007). However, there is limited research about multidigit arithmetic strategies used by lower achieving students, and if and how they differ from the normal and higher achieving students (Verschaffel et al., 2007). In this study, data from 685 Danish grade 6 students' written answers to multidigit addition, subtraction and multiplication was anlysed for accuracy and strategy type. In the written, Tri-phase Flexibility Assessment (Xu et al., 2017), students communicate their solution strategies. The strategies were categorised, number-based strategies, for example decomposition, sequential and compensation, digit-based strategies, most commonly the standard algorithm, no solution, when the student did not communicate a method, and other methods. The use of (i) number-based strategies and (ii) standard algorithm was compared by achievement group, as defined by results from the Danish national test in mathematics: Two low achieving student groups, LA1 (the lowest 10%), LA2 (next 25%), average achieving group, AA (the 30% of students closest to the mean score), and two high achieving groups, HA1 (the highest 10%) and HA2 (the next 25%).

ANALYSIS
Differences between achievement groups in use of calculation methods was analysed with ANOVA. Accuracy, measured as the proportion of tasks solved correctly with (i) number-based strategies and (ii) standard algorithm, was compared for different achievement levels with a t-test.

RESULTS AND PERSPECTIVES
The findings indicate that while number-based strategies were used increasingly with higher achievement level., the use of standard algorithms might be a result of sociocultural norms as no significant differences were found between different achievement levels.
For all achievement groups except LA1, number-based strategies were more accurate than standard algorithm. Differences in accuracy for the two lowest performing groups in number-based strategies, mean (SD) for LA1: 0.53 (0.40), and LA2: 0.80 (0.30), indicate that these two low performing groups have cognitively different challenges in mathematics.
The key findings of this study that the standard algorithm, although not a curriculum goal in Denmark, was found to be the most frequently used across all achievement levels, but with lower accuracy compared to number-based strategies, suggesting that all achievement levels would benefit from a more consistent focus on number-based strategies in arithmetic. The difference between the two lowest performing student groups further indicates, that the lowest performing 10% might need more targeted teaching in order to develop number understanding arithmetic ability.

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23 - LEARNING OPPORTUNITIES FOR STUDENTS ACHIEVING LOW IN MATHEMATICS

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The research project presented here is a multiple case study that addresses mathematical learning opportunities for low achieving students from a relational perspective. The research project was conducted in Norway from a mathematics education point of view. In line with Bagger et al. (2020), I recognize the importance of "for a diverse knowledge base in supporting students in need of support in mathematics" (p. 57).
Seven low achieving students (age 13–14) from different schools were observed and video recorded when working with higher achieving peers in heterogeneous groups on tasks that have low threshold and high ceiling (LTHC). Low threshold means that the task should be accessible to all students, all students should be in position to understand the question(s) posed. High ceiling means that the task can be solved in multiple ways, some more sophisticated than others. Therefore all students can solve the task with their preferred strategy and all students can get stuck.
In this research project communication is considered crucial for mathematical learning opportunities. Bakhtin (1981) has explained that dialogue and understanding go hand in hand: "Understanding comes to fruition only in the response. Understanding and response are diametrically merged and mutually condition each other; one is impossible without the other" (p. 282). Hence, the video recordings were analysed from a relational perspective. Further, the analytical process was inspired by Radford (2014) who emphasised humans' multimodal actions as the actualization of knowledge. Following Radford's work, the analysis focused on the students' multimodal actions in the heterogeneous groups. The analysis was conducted on two levels, first as case level analyses and then as a cross-case analysis.
The findings from the cross-case analysis identified some actions from peers that created opportunities for the low achieving students' actualization of mathematical knowledge and others that hindered such opportunities. Examples of actions that created opportunities are: requesting explanations, explaining own understanding, criticizing, and meeting unfamiliar concepts/ideas. Examples of actions that hindered opportunities are: blocking access to artefacts/manipulatives, ignoring ideas expressed by the low achieving students, and positioning the low achieving students as non-competente in mathematics.
However, the findings contribute with more than this. They illustrate that low achieving students can contribute substantially with mathematically sophisticated ideas when working in heterogneous groups. This aligns with findings from Barclay (2021), and I suggest that more research is needed on inclusive learning spaces.

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24 - ABOUT MATHEMATICS FOR STUDENTS WITH GENERAL DEVELOPMENTAL DISORDERS

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Background
In previous reports from the evaluation of special schools in Denmark, it appears that many schools in Denmark can find themselves marginalized in the municipal school management and development work and that help is needed to develop goals for students with multiple functional impairments and pervasive developmental disorders (Hedegaard-Sørensen & Tetler, 2014, p. 25) In England since 2007, P-scales are used in the teaching of pupils who work at levels below National Curriculum Level 1. P-scales have been developed in collaboration between researchers and special schools in England (QCA, 2017). P-scales have formed a background as a test tool. This has been widely criticized and in 2017, in England, they chose to make the work with P-scales optional. In Denmark, it has never been the intention that the material should be used in a test context, but rather as a tool for increased professional and didactic reflection on practice and progression in learning outcomes among students. It should be mentioned that the P-scale levels in England are still the common frame of reference for educational planning.

Purpose AND Question
The purpose of the project derived from a desire at four special schools to ensure and develop a high academic quality with a particular focus on Danish and mathematics. High academic quality means teaching that can be professionally justified, described, analyzed and assessed.
The formulation and development of the P-scales is intended to describe students' professional development in Danish and mathematics from 2 months to 6 years, which are the learning prerequisites on which Fælles Mål is based. It is also intended that the P-scales can contribute with a common language about the students' professional development across subjects in the work with these students.

The question investigated in the project was if and how P-scales could be a relevant tool for increased professional and didactic reflection on practice and progression, by
Describing students' professional development in Danish and mathematics from the age of 2 months to 6 years, which are seen as the learning prerequisites for the national Common Goals in Denmark for primary school.
Contributing with a common language about the students' professional development across subjects in the work with these students.

The P-scales are described in the table below.

P-skala	Dansk	Matematik
P1	Eleven møder verden og får oplevelser, uden tydelig læring. Eleven viser sporadisk interesse for aktiviteter og oplevelser og enhver respons fra eleven støttes.	
P2	Eleven responderer og deltager i kortvarig interaktion i kendte situationer. Eleven undersøger ting og reagerer ved at række ud og pege, på vej mod deltagelse.	
P3	Eleven deltager i aktiviteter med begyndende intentionalitet. Eleven involverer sig, og tager initiativ til kommunikation. Eleven begynder at forstå årsag/virkning sammenhænge.	
	Dansk	Matematik

P4 P5 P6 P7 P8	Kommunikation – lytte og forstå	Kommunikation – Tale og udtrykke	Sproglig opmærksomhed	Funktionel matematik	Tal og antal	Figurer, mønstre og måling
Fælles Mål	Dansk			Matematik		

Table 1: The P-scale

Methodology
Action learning was chosen as method in order to contribute to the development of new knowledge and implement change in practice (Plauborg et al., 2007), through: development of theoretical knowledge, including the operationalization of academic goals in Danish and mathematics for special schools that work at competence levels before the national Common Goals in Denmark for primary school.
qualifying teaching through critical reflection.
contributing to change through competence development.
contributing to development in the overall organization through collaboration between four school leaders, the municipality of Copenhagen and the schools' teachers and pedagogues.

Participants
Four category 4 schools in the municipality of Copenhagen participated in the project: Fensmarskolen, Heerupsskole (School in Ryparken), Øresundsskolen and Strandparkskolen. Category 4 schools are characterized by being schools for students with general developmental disorders, where the child's entire profile of cognitive, language and motor development areas is severely delayed compared to normal development (Trillingsgaard, 2003).

Results
In May 2018, we carried out a qualitative evaluation of the project group participants' perception of five aspects of the project. The participants were asked for their views on what will benefit the purpose of the project in the future and what changes the project has brought about in the schools so far: The evaluation was carried out with inspiration from the Delphi evaluation method. In summary, the following points emerged as central in the evaluation.
Participants expressed their clear perception that P-scales must be used in student plans to create change
Participants expressed that a common language including P-scales about professional development for the student group is well under way.
Participants expressed a desire for continued and more team collaboration and professional learning communities.

Literature
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25 - How do students explain and understand different variables?

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Working with variables in mathematics can be challenging for students in mathematical difficulties. However, teachers may find it hard to understand why students have difficulties working with variables, and often students with difficulties cannot explain what it is that they find challenging. The aim of the present study was to determine how students explain and elaborate on their understanding of variables. Thus, the overarching research question was "How do students explain and elaborate on their understanding of variables?"

Procedure
The present study was based on two teacher-students' research work for their bachelor's thesis and the contributions of their counsellor to it. Five semi-structured interviews were conducted with five participating ninth-grade students. The students were selected by their regular mathematics teacher based on their good ability to communicate and explain their mathematical ideas. The interview guide was developed from international research and was evaluated by researchers connected to VIA University College. As indicated in the guide, every question was based on a mathematical task with a solution. The students were asked to elaborate on whether the solution made sense and could be explained.

Results
A central finding was the student's answers to the question where two different variables represented the same value (i.e., What does the expression y = x mean?) One student answered, "The expression means that y is equal to x; that is, if y is 1, x will be 3, and if y is 2, then x will be 6." The student explained that in the given expression, the different letters have different values. She argued that the expression 2 = 6 based on her understanding of the names on the variables are different. This shows that the student's misconception or lack of understanding of the equal sign also needs to be addressed—that is, the student needs to develop the understanding of the symmetric property (e.g., a = b and b = a). Overall, we found that the equal sign plays an important role in working with variables and that the students' difficulties in this task were often based on their difficulties in understanding different mathematical concepts that are connected to variables.

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