Difficult numbers?

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Are numbers perceived differently?

Does 87 feel worse than 94?

And does it feel even worse if you see yourself as bad at math?

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Pause a moment!

Couldn't it be the exact opposite?
The more you see yourself as bad at math the more indifferent the numbers should be – all numbers are bad!

We saw something like that in word problems:
Low performers only recognize straightforward addition word problems – the rest are all the same, they are hard!
(Pind et al., 2021 presentation at NORSMA 10)

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Background

Based on analysis of correctness:

Children find single-digit multiplication with operands 7 and 8 more difficult than multiplications with other operands (Taraghi et al., 2014; van der Ven et al., 2015).

Differences in difficulty levels have been explained by several aspects, for example problem-size effect (for an overview see van der Ven et al., 2015).

How the difficulty of numbers is perceived irrespectively of the individuals' ability to perform a calculation correctly, has to our knowledge not been systematically investigated.

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This study

We present here a first exploratory study investigating the research questions:

Are some numbers perceived as more difficult than others?

If so, is this dependent on general mathematical ability?

This study

Facebook (not a random sample!)
Adults age >18
691 adults: 515 women, 174 men

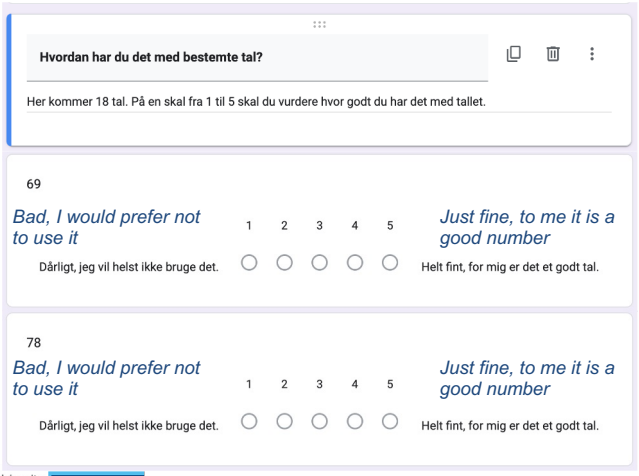
Self-report on age, gender, and mathematical ability (1 to 5)

Number preferences, multiplications and calculations in context

Self-report on preferences and likelihood of success on a scale from 1 to 5

An open question on experiences with numbers in general

Examples, Number preferences



How do you feel about certain numbers?

We will show you 18 numbers. On a scale from 1 to 5, please rate how you feel about the number.

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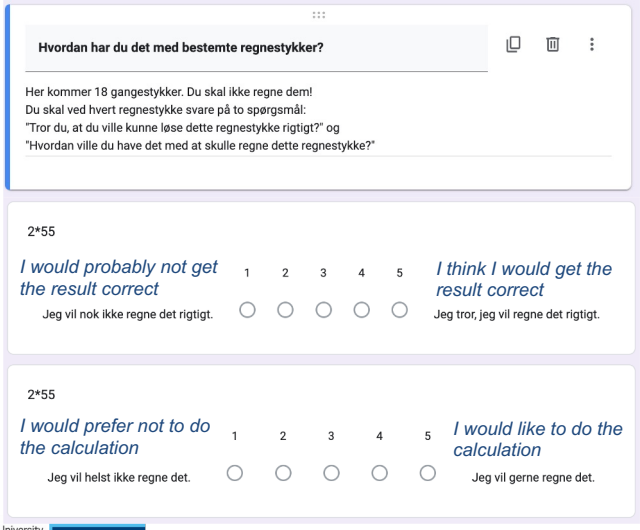
Examples, Multiplications



How do you feel about certain calculations?

We will show you18 multiplications. Don't solve them!

For each calculation, we ask you to answer two questions :

"Do you think you would be able to solve this math problem correctly?"

and "How would you feel about having to do this calculation?"

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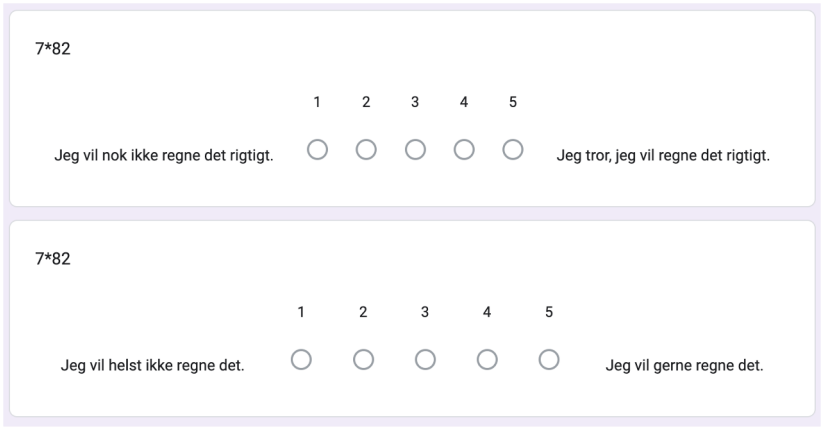
Examples, More multiplications



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Examples, Calculations in context

The price difference for diesel and petrol is the same in the two cities of Duckburg and Goosetown.

In Duckburg, a liter of diesel costs DKK 17.78 and a liter of petrol DKK 18.57.
In Goosetown, a liter of diesel costs DKK 15.43 and a liter of petrol DKK 16.22.

In which city would you prefer to calculate the price difference?

- ❑ Duckburg
- ❑ Goosetown
- ❑ I think it is equally easy or difficult

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Analysis

Item difficulty was scored based on difficulty of included digits:

(*e*) easiest: 1, 2, 5 and 10
(*i*) intermediate difficult: 3, 4, 6, 9,
(*d*) most difficult: 7 and 8.
following the findings of Taraghi et al. (2014)

Score	2-digit items	Example	3-digit items	Example
1	ee	2 * 5; 25	eee	2 * 55; 255
2	ei	4 * 5; 41	eei	2 * 53; 451
3	ed	5 * 7; 27	eed	5 * 57; 272
4	ii	6 * 9; 69	eii	6 * 92; 692
5	id	3 * 8; 89	eid	8 * 92; 382
	dd	7 * 8; 78	edd	7 * 82; 782

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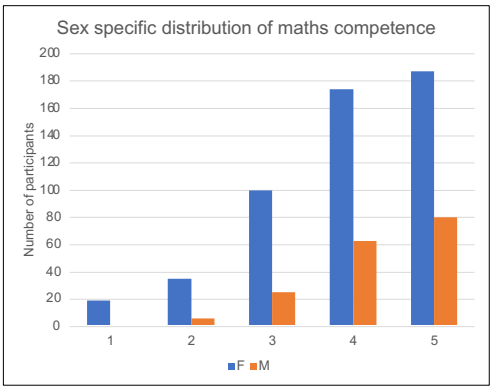
Results:
Distribution of maths competence

The sample:
Facebook (not a random sample!)
Adults age >18
691 adults: 515 women, 174 men

Self-report on age, gender, and mathematical ability (1 to 5)

Mean scores:
Females: 3.92
Males: 4.25
Difference = 0.32 ($p < 0.0001$)



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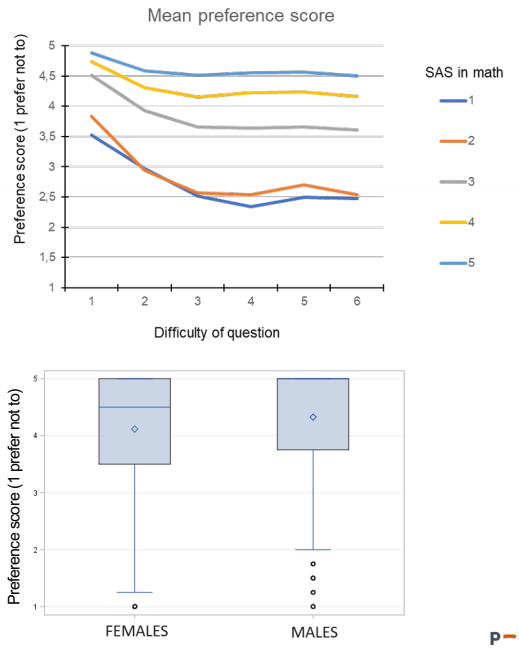
Preference score

GLM (type-3 analysis):
Difficulty of question (DQ): $p < 0.0001$
Self-assessment score (SAS): $p < 0.0001$
Interaction btw. DQ*SAS: $p < 0.0001$

Sex $p < 0.0001$

	mean
Females	4.12
Males	4.33
Diff. (raw)	-0.21
Diff. (partial)	-0.05



Results

Overall, men reported all numbers as easier than women even after correcting for self-evaluated mathematics ability level and difficulty of numbers.

Low ability groups perceived difficult numbers (numbers including digits 7 and 8) as more difficult than high ability groups did. The differences in scores indicate that low achievers perceived difficult numbers as relatively more difficult than high achievers did.

Possible explanations

Could it be that in our base 10:

- the easiest are the factors: 1, 2, 5, 10
- the intermediate are neighbours: 3, 4, 6, 9
- the most difficult are the remaining: 7 and 8

Thumb is the most extreme here, therefore the easiest.

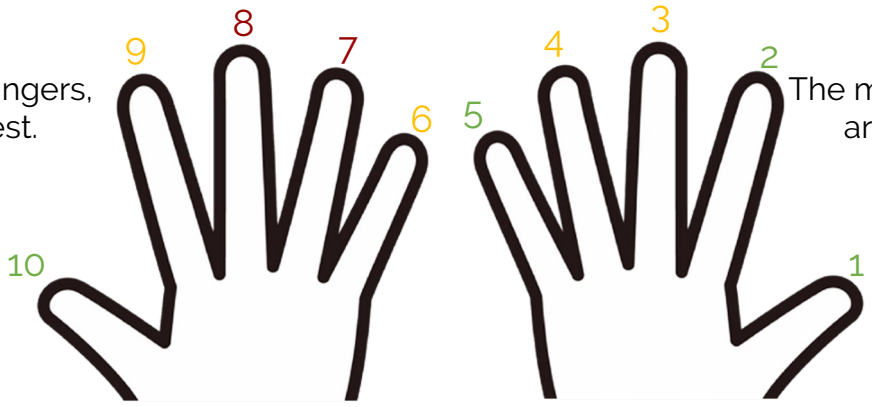
Little finger, and index finger are next-easiest.

The middle fingers, are the hardest.

Right hand numbers are smallest - therefore generally easier than left.

Extreme fingers and index finger easiest.

The middle fingers, are the hardest.



Possible explanations

For a right-handed person :-)

Possible consequences

"Beliefs of personal efficacy constitute the key factor of human agency,"…"If people believe they have no power to produce results, they will not attempt to make things happen." (Bandura, 1997)

There is a risk that (self-perceived) low performers will not engage in problem solving including numbers they perceive as difficult.

As a result, they will have less experience and fewer potential successes in problem solving further contributing to low self-efficacy.

Status quo in exams

Danish 9th grade exam, without calculator:

Opgave 5

Regn stykkerne.

5.1	1072 + 4038	=
5.2	701 – 149	=
5.3	350 · 9	=
5.4	7021 : 7	=

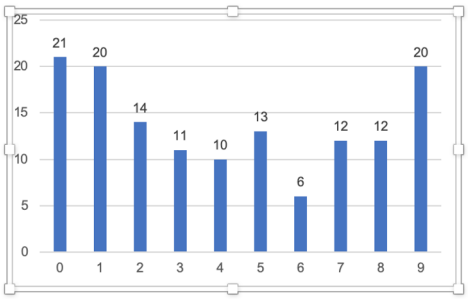
may 2019

These 4 types of problems each year.

Status quo in exams

Analysis of digits.

From maj 2019 to maj 2023, 6 exams:



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Status quo

Is difference in digit difficulty unconscious common knowledge?

Do we already use certain digits to “spice it up or cool it down”?

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Implications

Does it matter?

Implications in Exams

5.4 7021 : 7 = _____ *may 2019*

7021:7 =

31% of students did not get the right answer.

What would it be, if it was 5015:5=?

5.4 15030 : 15 = _____ *may 2023*

15030:15=

32% of students did not get the right answer.

What would it be, if it was 17034:17=?

Implications in Exams

Did the exam makers spice up the first one and cool of the second one?

You could say they are pretty good at hitting around 30% failure :-)

Implications

What would exams and textbooks look like if it was common conscious knowledge, that some digits are perceived as more difficult for students with math anxiety and/or students with math learning difficulties?

Would we be more aware not to let the numbers be a barrier for math learning?

Next round in study

What should we do differently?

- we know, we would not have single digit multiplications
- include subtraction?
- test ability?
- ask to perform calculations?
- more word problems?
- include children?

REFERENCES

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