

What is the ‘Work’ of Mathematics Teaching, and How Can Research Contribute to It?

Deborah Loewenberg Ball

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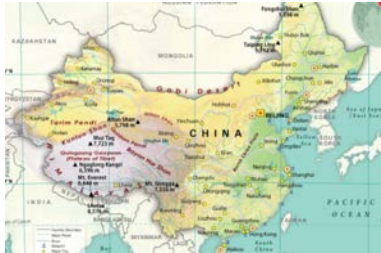




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Teaching is the largest occupation in any country.



15 million teachers



3.8 million teachers



1 million teachers

72.5 million teachers worldwide



41,900 teachers



410,000 teachers



1.4 million teachers



8.7 million teachers

**We depend on teaching and teachers to
develop our most valuable resource.**

Our young people.



What do we want from mathematics education?



Society and economy:
to build a skilled
technical workforce

Justice: to disrupt patterns that push
out students from marginalized groups

What do we want from mathematics education?

Literacy: to grow a
generation of adults who do
not reject mathematics

The field of 'maths': to expand
conceptions of mathematics and
mathematical competence

Society and economy:
to build a skilled
technical workforce

Justice: to disrupt patterns that push
out students from marginalized groups

What do we want from mathematics education?

to advance young people's
mathematics flourishing

Literacy: to grow a
generation of adults who do
not reject mathematics

The field of 'maths': to expand
conceptions of mathematics and
mathematical competence

mathematical flourishing

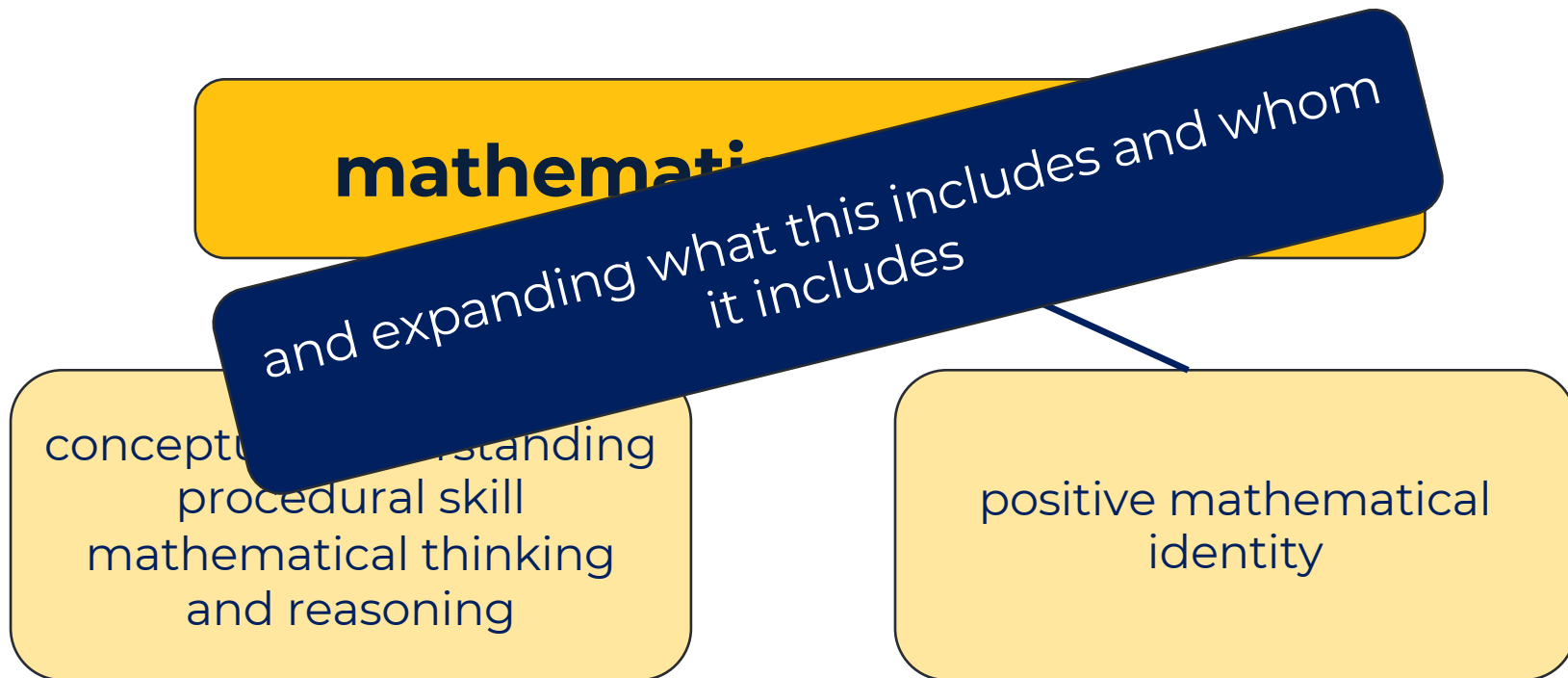


mathematical flourishing

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graph TD; A[mathematical flourishing] --> B[conceptual understanding<br/>procedural skill<br/>mathematical thinking<br/>and reasoning]; A --> C[positive mathematical<br/>identity]
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conceptual understanding
procedural skill
mathematical thinking
and reasoning

positive mathematical
identity



These are not new goals.

But after many decades of cycles of ‘reform,’ most mathematics classrooms have not changed much.



Teachers explain how to “do” procedures.

Getting the answer right is the goal.

“Application” problems often seem irrelevant or meaningless to students.

Some students think they are “good at math” while others think they are bad at it.

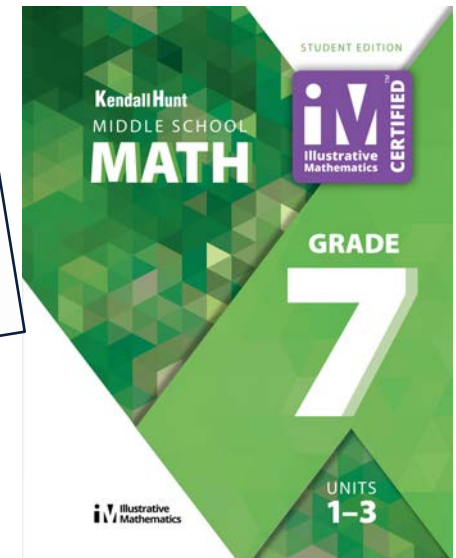
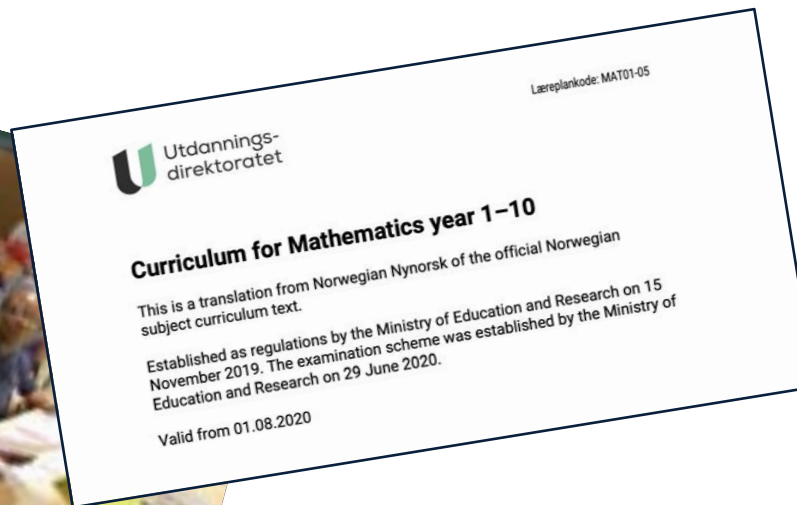
Young people who are members of marginalized groups are pushed out.

Students are sorted by judgments about their mathematical “ability.”

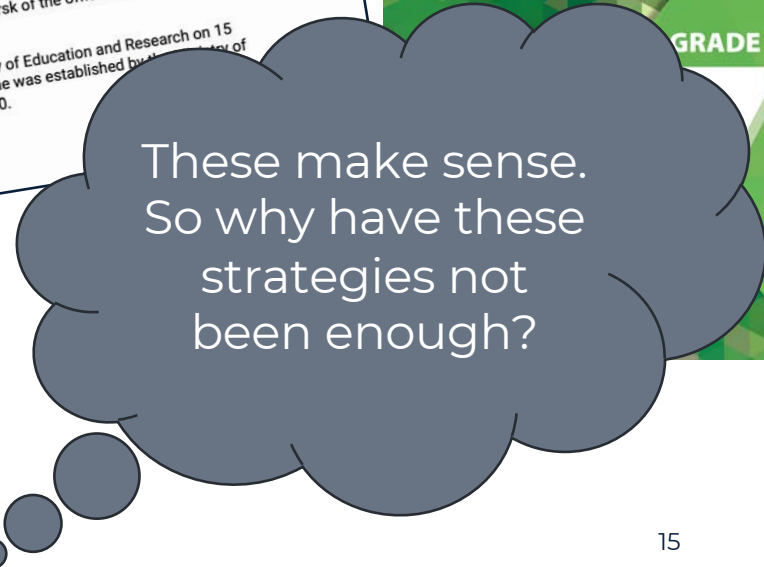
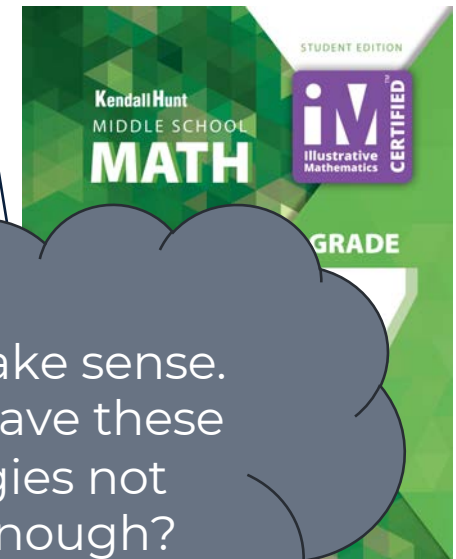
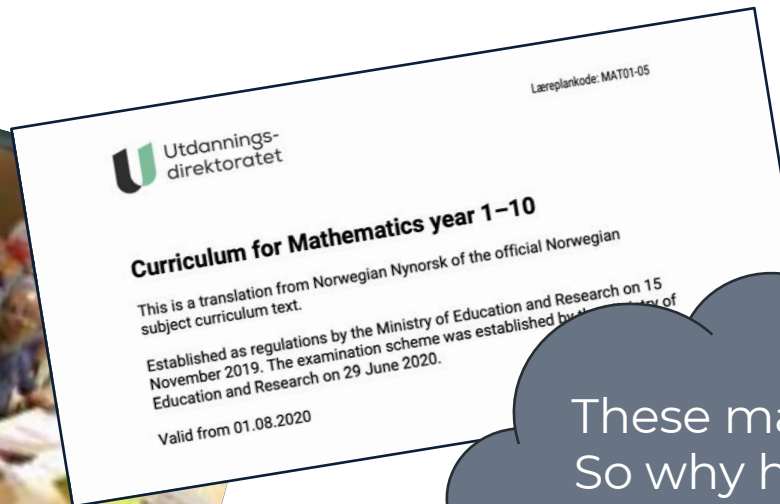
Meanwhile, arguments and polarized discourses persist.



What are the common strategies that keep being used to improve young people's mathematical flourishing?



What are the common strategies that keep being used to improve young people's mathematical flourishing?



Today's argument

We lack sufficient understanding of **the work of teaching** — and its relation to mathematics learning.



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Why do I use this phrase ‘work’ of teaching? And what does it mean?

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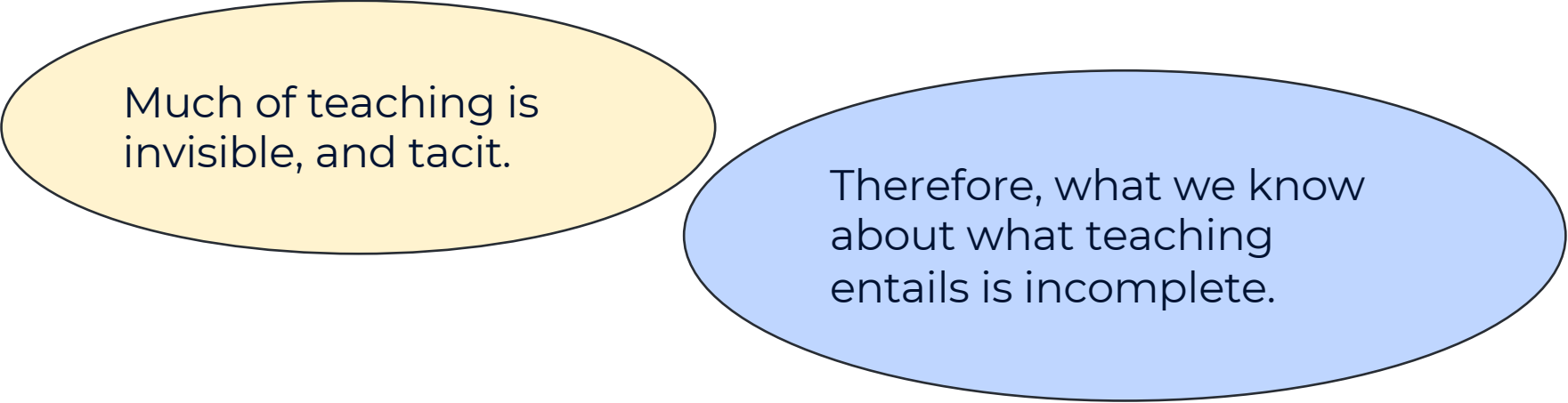


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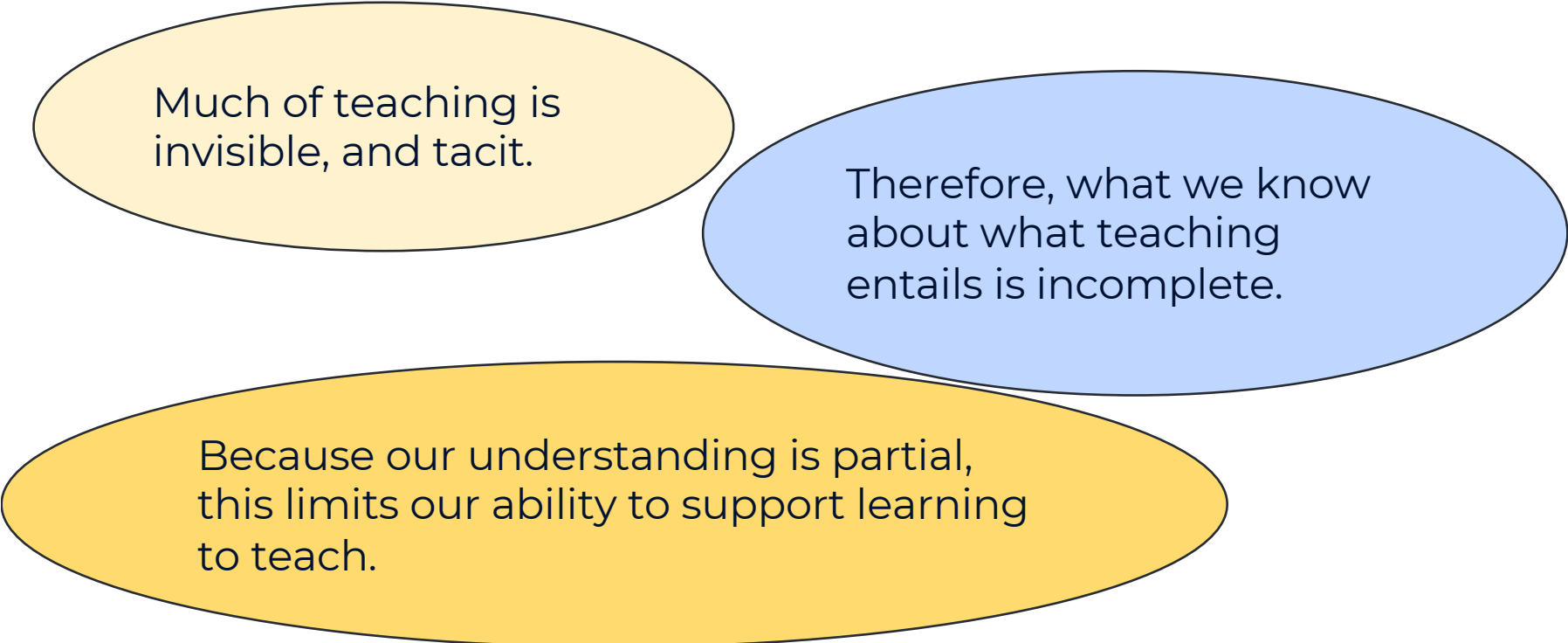
Much of teaching is
invisible, and tacit.





Much of teaching is
invisible, and tacit.

Therefore, what we know
about what teaching
entails is incomplete.

The image features three overlapping ovals. The top-left oval is light yellow and contains the text 'Much of teaching is invisible, and tacit.' The top-right oval is light blue and contains the text 'Therefore, what we know about what teaching entails is incomplete.' The bottom oval is a darker yellow and contains the text 'Because our understanding is partial, this limits our ability to support learning to teach.'

Much of teaching is invisible, and tacit.

Therefore, what we know about what teaching entails is incomplete.

Because our understanding is partial, this limits our ability to support learning to teach.

Much of teaching is invisible, and tacit.

Therefore, what we know about what teaching entails is incomplete.

Because our understanding is partial, this limits our ability to support learning to teach.

And reform efforts too often fall short.

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**To improve young people's mathematical flourishing
in our countries, we need to understand more about
the work of teaching.**



We need more focus on the
work of teaching mathematics.



Using and adapting curriculum materials
Creating a classroom environment for learning
Assessing students' learning
Communicating with families

We need more focus on the
work of teaching mathematics.

Connecting mathematics to students' contexts
Knowing and using mathematical knowledge



Using and adapting curriculum materials
Creating a classroom environment for learning
Assessing students' learning
Communicating with families

We need more focus on the
work of teaching mathematics.

And understanding the density of discretionary spaces inside the work.

Connecting mathematics to students' contexts
Knowing and using mathematical knowledge

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The claim

We need more research that focuses inside the nuances of actual practice.

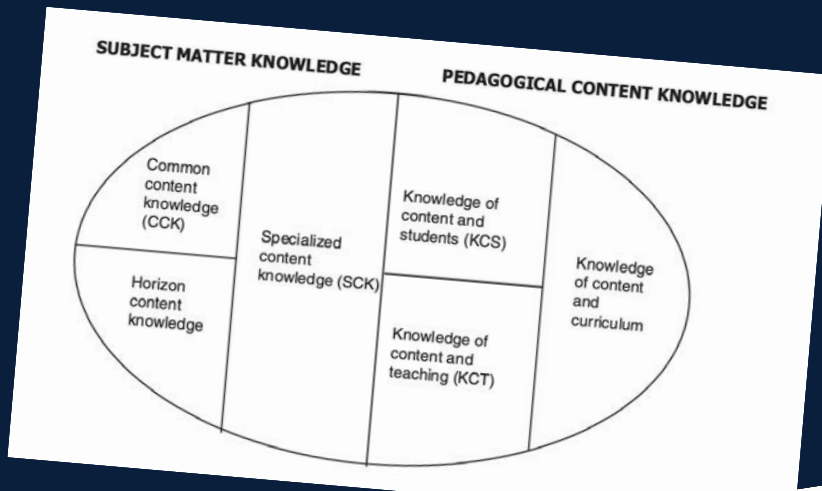
1. Fostering young people's mathematical flourishing is essential.
2. Many efforts have been made over decades to do this.
3. Still not a lot has changed inside mathematics classrooms, achievement is flat, and patterns of exclusion and inequity persist.
4. The failure to understand the work of teaching is a foundational flaw.



The Claim

We need more research that focuses inside the nuances of actual practice.

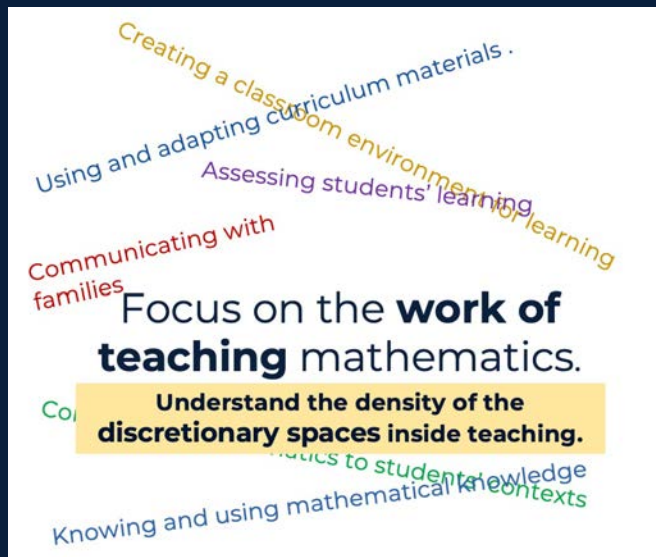




The Unrealized Promise of High-Quality Instructional Materials
Overcoming barriers to faithful implementation requires changing teacher and leader mind-sets.
By David Stinner

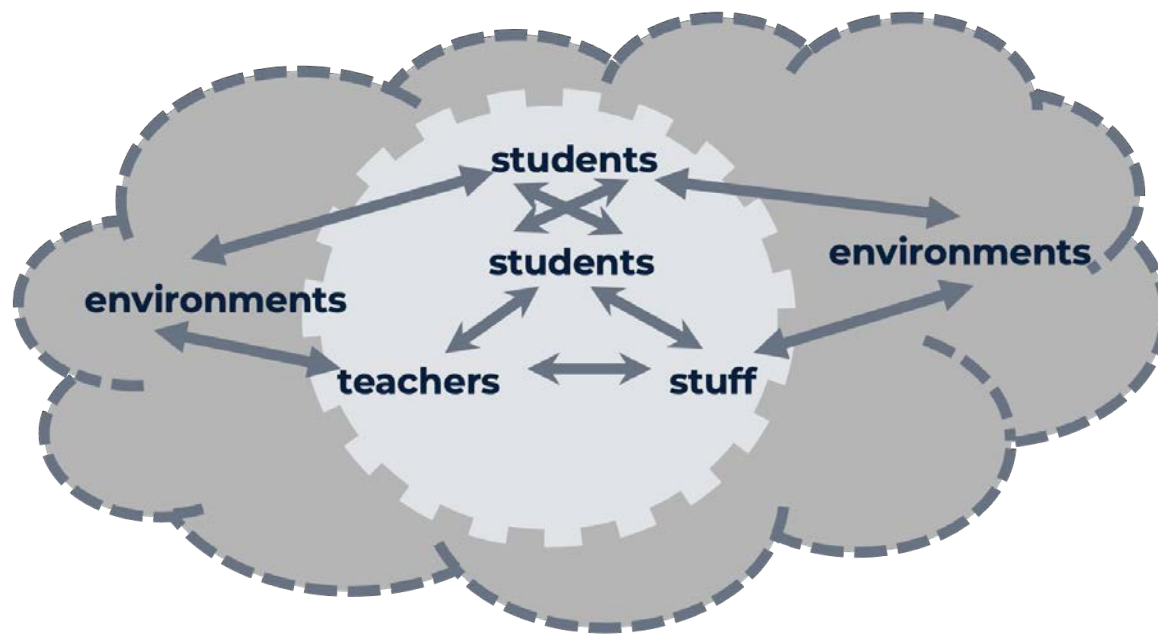
Two theorems

1. Mathematical knowledge for teaching (MKT) is necessary but insufficient for improved mathematics teaching.
2. Good instructional materials matter but materials do not by themselves teach.

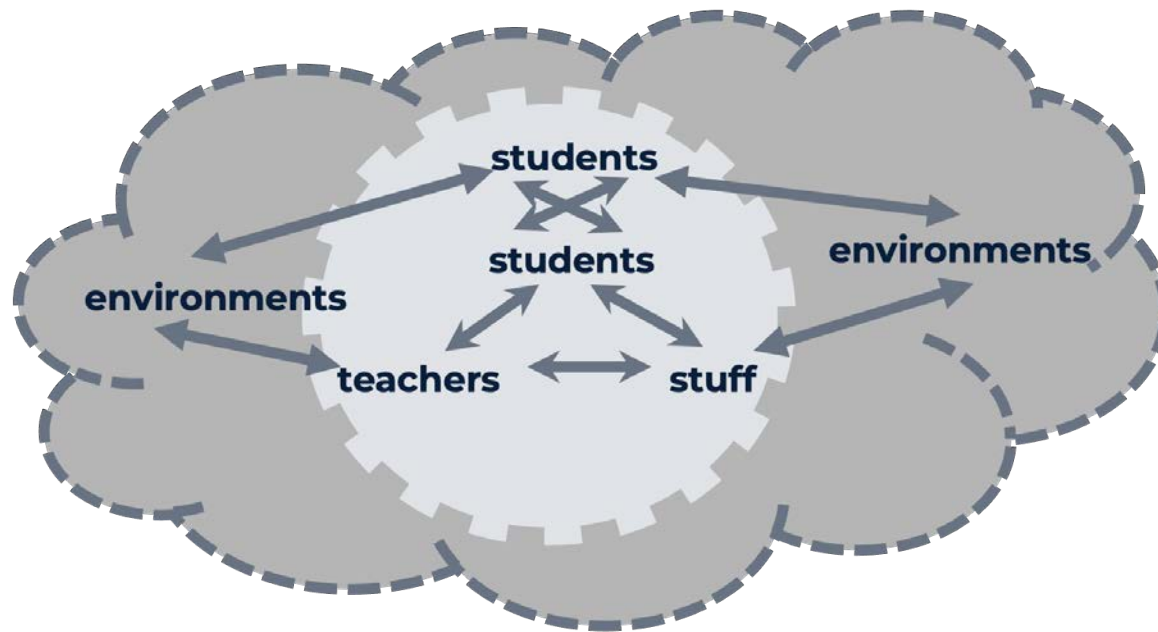


Two lemmas

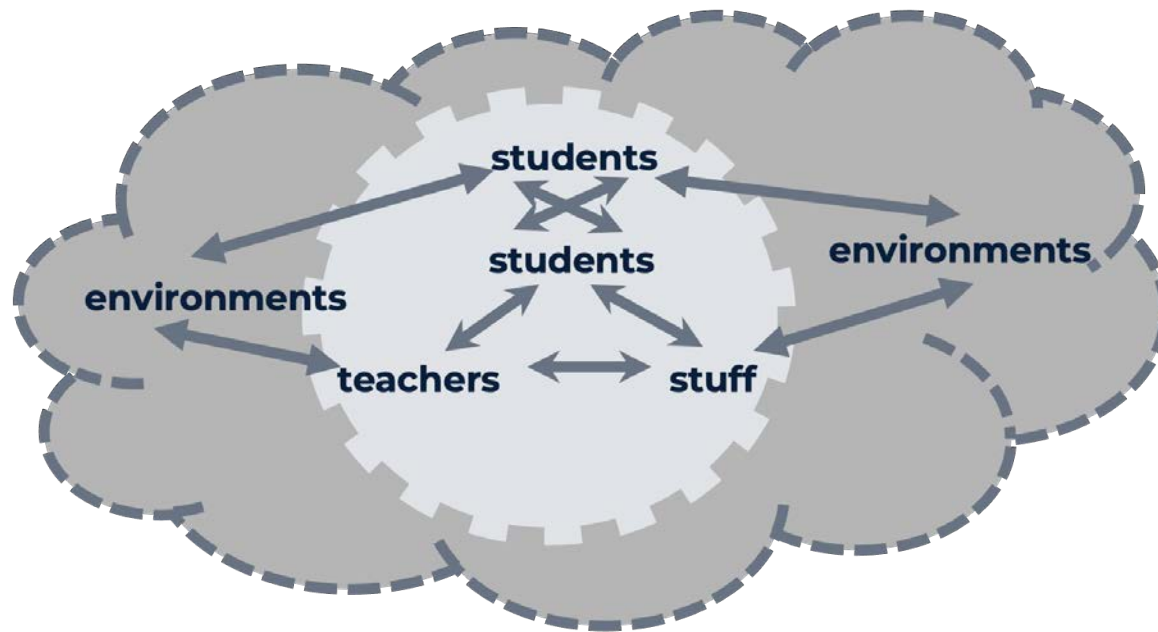
1. The work of teaching mathematics defies dichotomies.
2. The work of teaching mathematics inherently entails tremendous discretion.



1. Teaching and learning are constructed interactively and are interpretive in context.



1. Teaching and learning are constructed interactively and are interpretive in context.
2. They take place within broad historical and socio-political environments.



1. **Teaching is powerful.** When it is done with care and judgment, students can thrive — learn mathematics, develop positive identities, learn to value others and work collectively.
2. Teaching also involves **enormous discretion.**
3. How that discretion is exercised can **cause harm, or it can support** young people's mathematical flourishing.

The Proof

Studying the case of Antar, Gabriella, Gabi, Virshawn, Marquis, Kassie, and their classmates



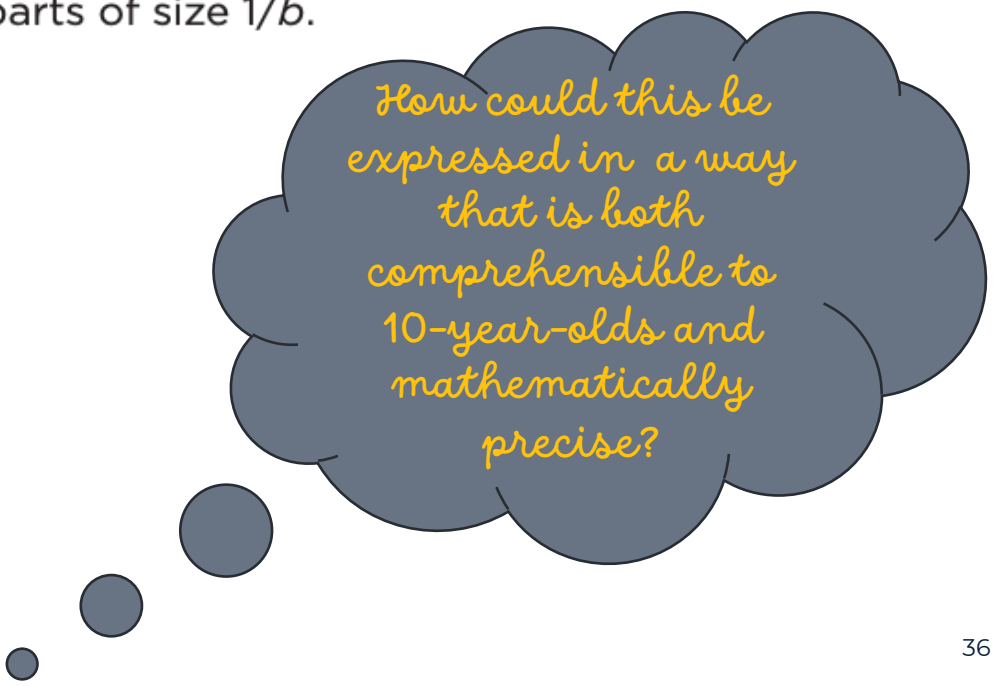
Fractions: The given instructional goal

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



Fractions: The given instructional goal

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



How could this be expressed in a way that is both comprehensible to 10-year-olds and mathematically precise?

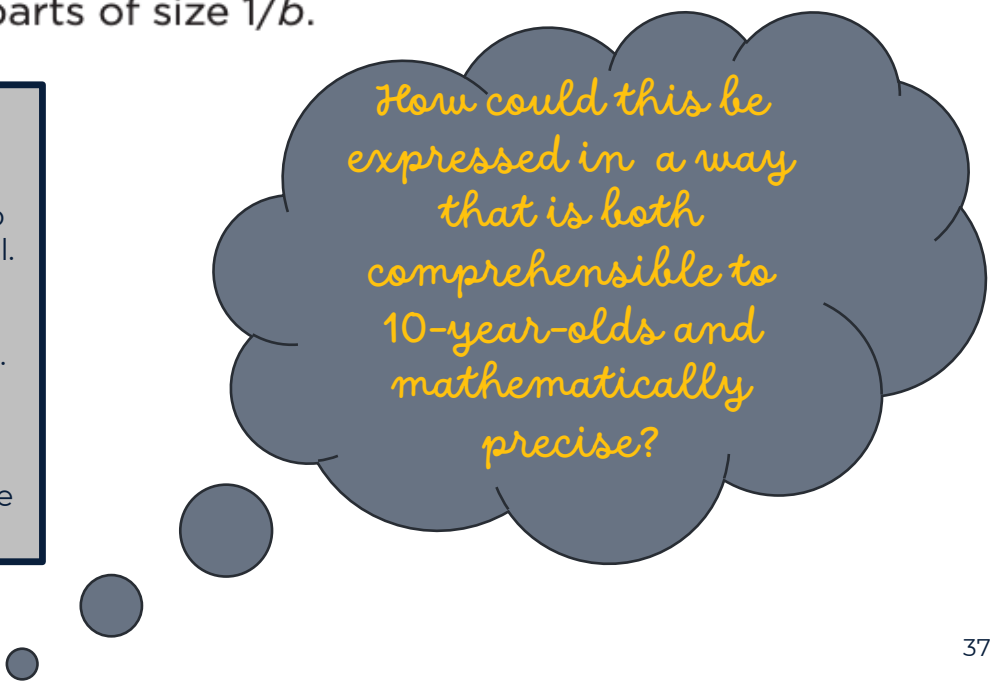
Fractions: The given instructional goal

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NAMING A FRACTION

1. Figure out what the whole is.
2. Make sure that the whole is divided into equal parts. If not, make the parts equal.
3. Count how many equal parts there are. Call this number d .
4. Write $1/d$ to show one of the equal parts.
5. If more than 1 of those parts is shaded, count them (n) and write (n/d) .

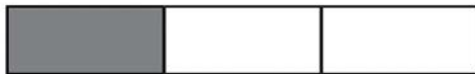
NOTE: $d \neq 0$ and (in fourth grade) n must be a whole number.



How could this be expressed in a way that is both comprehensible to 10-year-olds and mathematically precise?

The given mathematics task from a “high-quality instructional material”

2. Juanita cut her string cheese into equal pieces as shown in the rectangles below. In the blanks below, name the fraction of the string cheese represented by the shaded part.

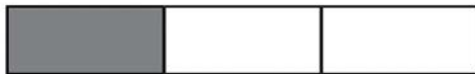






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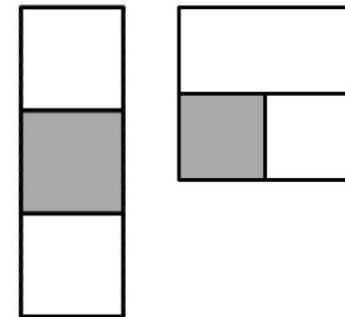






An adapted mathematics task

What fraction of each rectangle below is shaded gray?



The given mathematics task from a “high-quality instructional material”

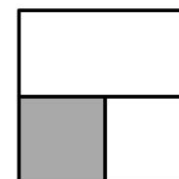
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*Both align
with the
learning goal.
What is the
same and what
is different?*

An adapted mathematics task

What fraction of each rectangle below is shaded gray?



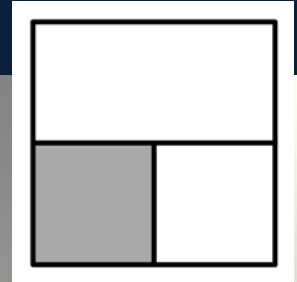
First Viewing

We will see and hear three of the children: Antar, Gabriella, and Gabi.

What do you see about the work of teaching mathematics in this video clip?



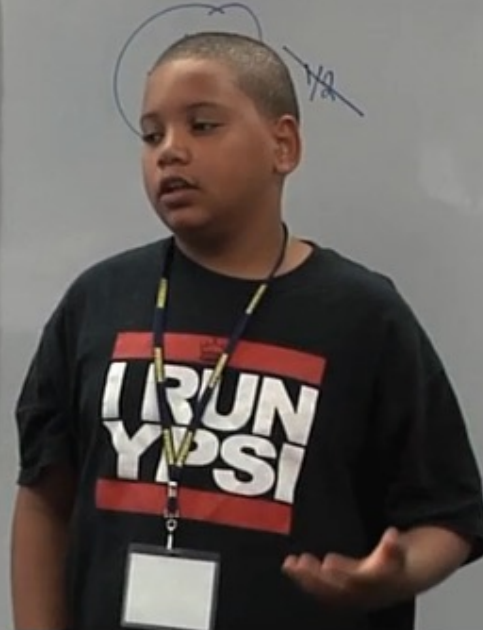
Video: Antar, Gabriella, and Gabi



What fraction of the rectangle below is shaded gray?



What fraction of the rectangle below is shaded gray?



Antar: I think it's not a fraction because all the parts are not equally the same.



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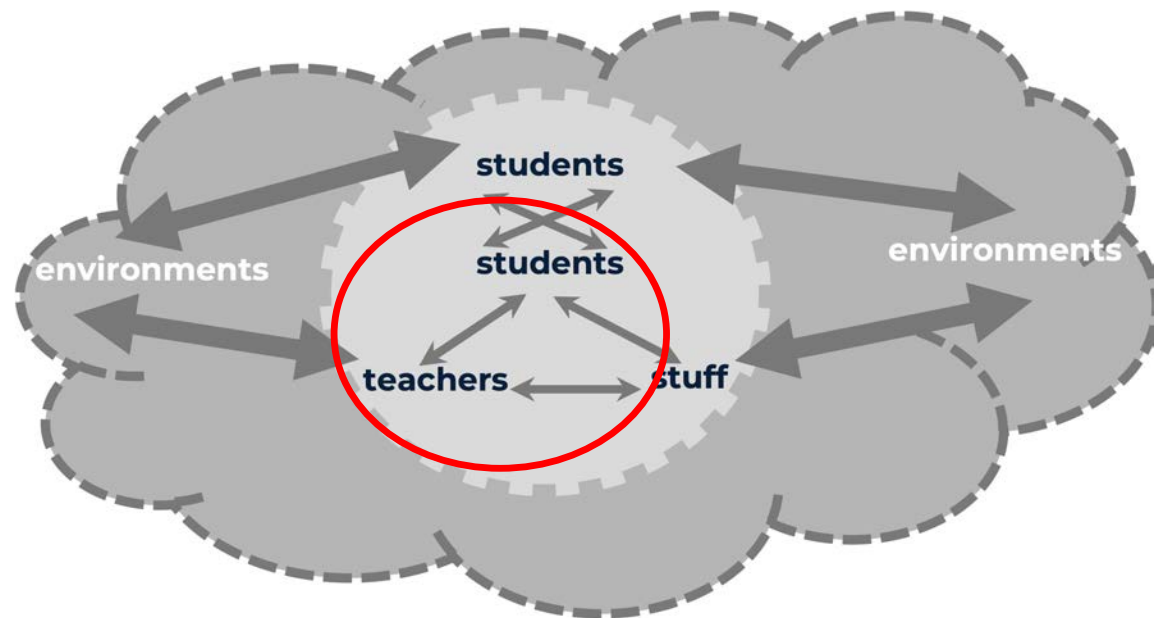
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First Viewing

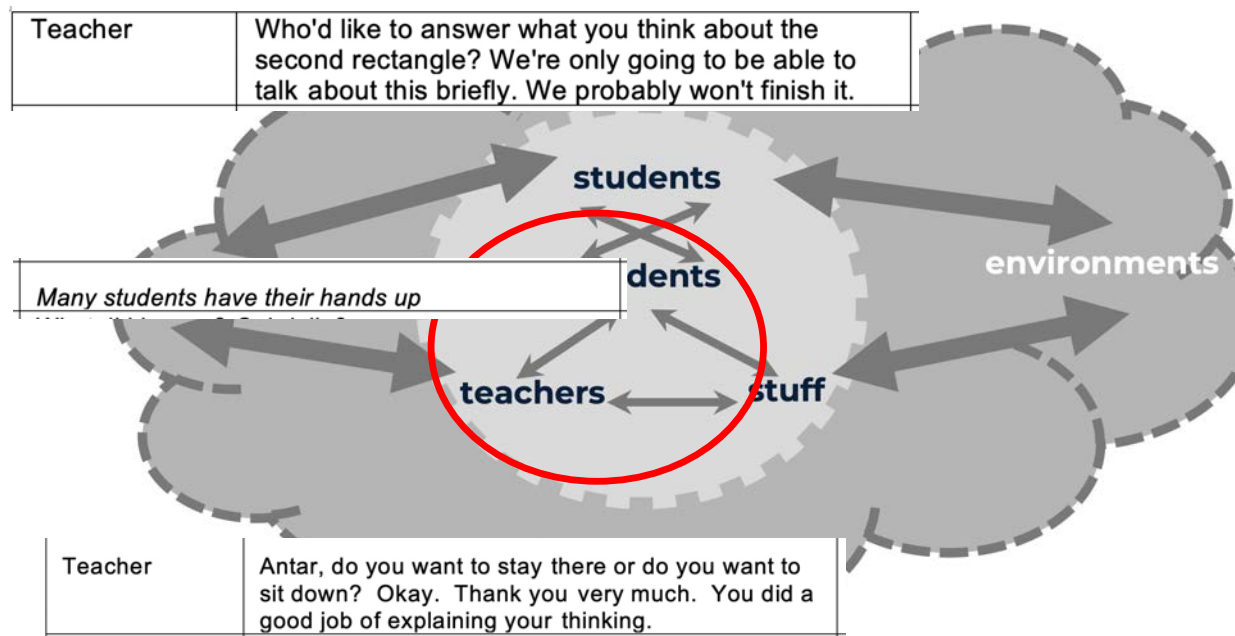
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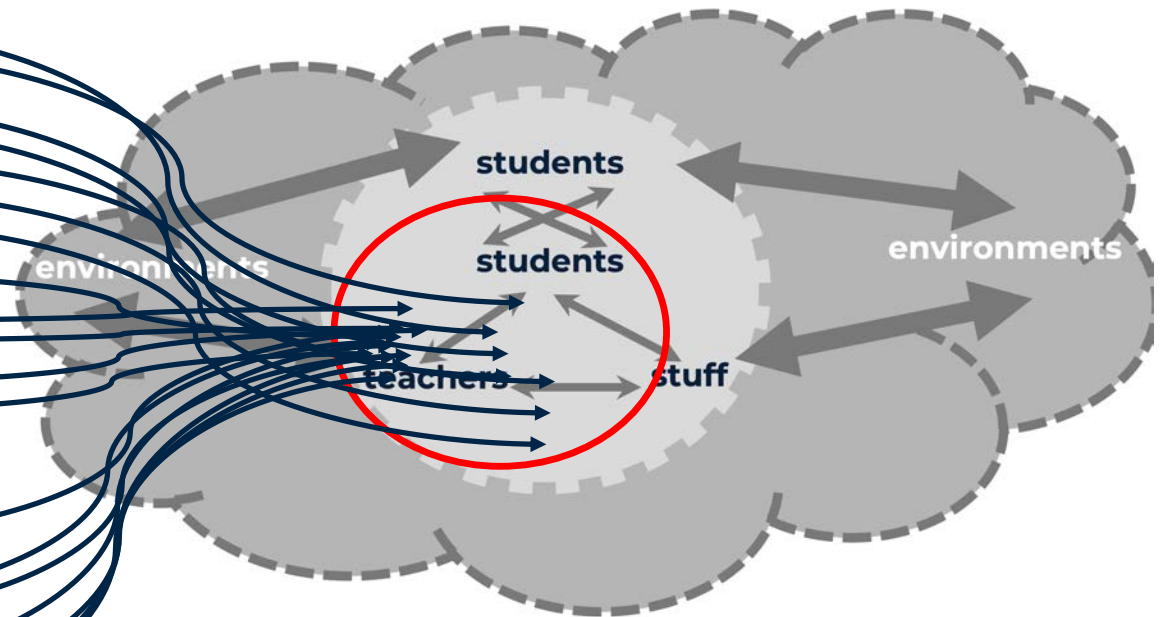
Teacher	Who'd like to answer what you think about the second rectangle? We're only going to be able to talk about this briefly. We probably won't finish it.	Launch discussion
Teacher	Who'd like to explain what you think? Antar, what do you think?	Choose student to call on
Teacher	Could you come up to the board and explain? Thank you.	Frame task for student who is presenting
Teacher	I really like the way that people who are coming to the board are doing today. You are explaining really well.	Acknowledge competence
Teacher	Here's a marker. Can you explain your thinking?	Provide material support
Antar	I think it's not a fraction because all of the parts are not equally the same shape.	Listen
Teacher	Can you say that one more time to the class?	Support presenter
Antar	I think it's not a fraction because all the parts are not equally the same.	Listen
Teacher	Can someone repeat what Antar said? Very nice, Antar.	Orient students to presenter
	Many students have their hands up	
Teacher	What did he say? Gabriella?	Choose student to call on
Gabriella	Oh. He said that he doesn't think it's a fraction because not all the parts are equal.	Listen
Teacher	Is that what you said?	Position first student as authority
Teacher	Okay, would someone like to comment on that? Agree or disagree with him?	Orient students to one another
Teacher	Okay, let's see, how about Gabi.	Choose student to call on
Gabi	I disagree.	Listen
Teacher	What do you think?	Pose question
Gabi	I think the fraction is one-fourth.	Listen
Teacher	One-fourth? Do you want to come up and say why you think it's one fourth?	Frame next step, support next presenter
Teacher	Antar, do you want to stay there or do you want to sit down? Okay. Thank you very much. You did a good job of explaining your thinking.	Position student with agency, acknowledge competence
Teacher	So, let's hear what Gabi's thinking.	Orient students to one another
Gabi	I think it's one-fourth because, like he said, all the fractions aren't the same, but you can make them the same by dividing a line down the middle.	Listen
Teacher	Here's something you can use so if someone wants to take it off again, they can. Okay, so now explain what you've done. Talk to the class, okay?	Provide material support
Gabi	I divided it down the middle because, since it's not equal, you have to make it equal.	Listen
Teacher	And so then you decided?	Probe
Gabi	It's one-fourth.	Listen



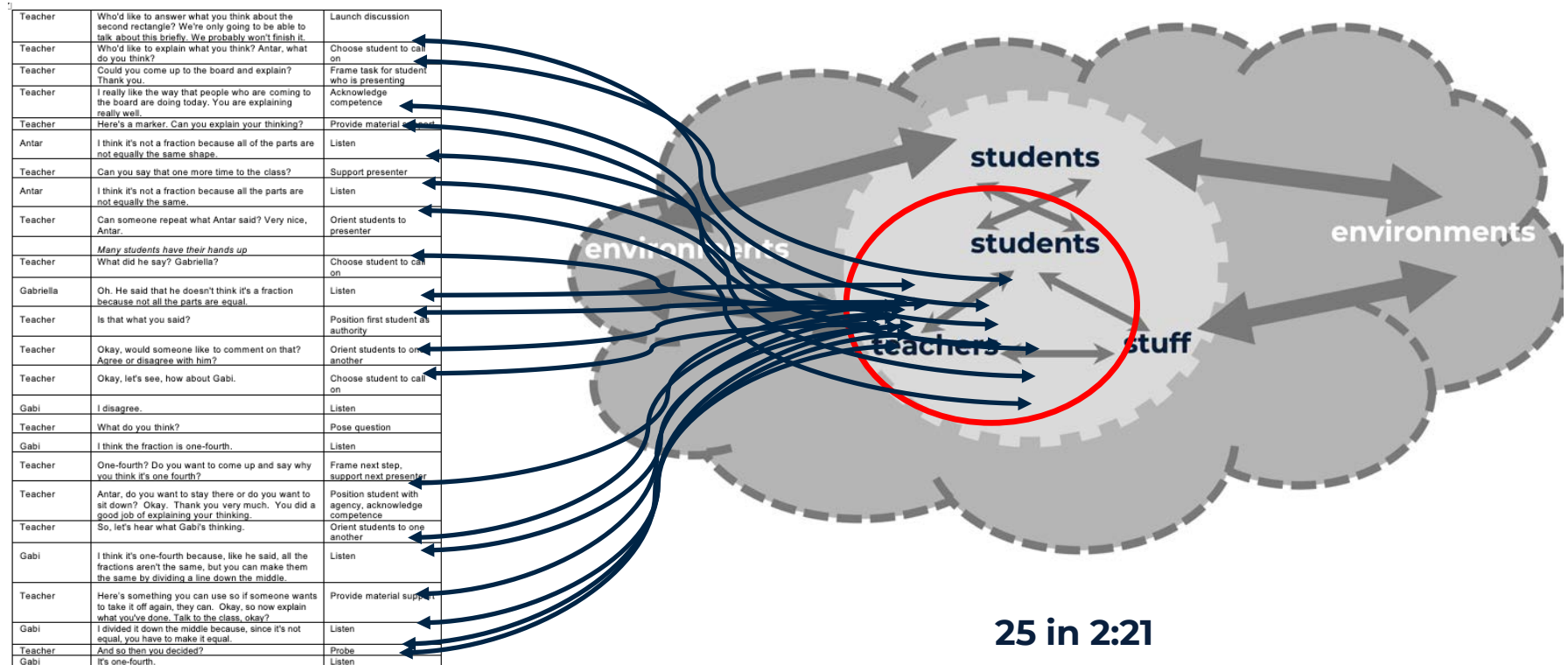
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Teaching is dense with “discretionary spaces”



(Some of) the teaching and learning goals:

- Helping students understand fractions
- Supporting collective mathematical work
- Supporting mathematical listening, appraising validity, questioning
- Disrupting patterns of what is seen as mathematical competence and who is seen as mathematically competent
- Supporting the development of positive mathematical identities



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Second Viewing

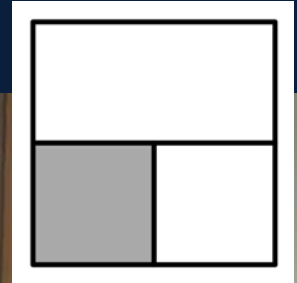
This time, we will see and hear six of the children: again, Antar, Gabriella, and Gabi, but also Virshawn, Marquis, and Kassie.

What discretionary spaces do you see in the work of teaching mathematics in this video clip?

- Things that the teacher does
- Things the teacher does not do



Video: Antar, Gabriella, Gabi, Virshawyn, Marquis, and Kassie



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Second Viewing

What discretionary spaces did you see in the work of teaching mathematics in this video clip?

- Things that the teacher does
- Things the teacher does not do



A few discretionary spaces in the clip

- What to do after Antar explains, 'It's not a fraction because the parts are not equal.'
- Where to stand in the room
- What to do after Gabriella restates Antar's explanation
- Whom to call on
- What to do about Antar at the board
- What to do when Virshawn cannot say what Gabi said
- How to respond to Marquis when he holds up his notebook and says what Gabi explained
- What to say after Kassie shows why she disagrees with Gabi



A few discretionary spaces in the clip

- What to do after Antar explains, 'It's not a fraction because the parts are not the same shape.'

- Where to stand in the room
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- Whom to call on after Gabriella

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- What to do when Virshawn cannot say what Gabi said

- How to respond to Marquis when he holds up his notebook and says what Gabi explained

- What to say after Kassie shows why she disagrees with Gabi

Some possible next moves

- Say, 'It is a fraction. Can you make the parts equal?'
- Ask, 'Who can help Antar?'
- Ask, 'Who can say what Antar said?'

A few discretionary spaces in the clip

- What to do after Antar explains, 'It's not a fraction because the parts are not the same shape.'

- Where to stand in the room
- What to do after Gabriella restates Antar's explanation

- Whom to call on after Gabriella

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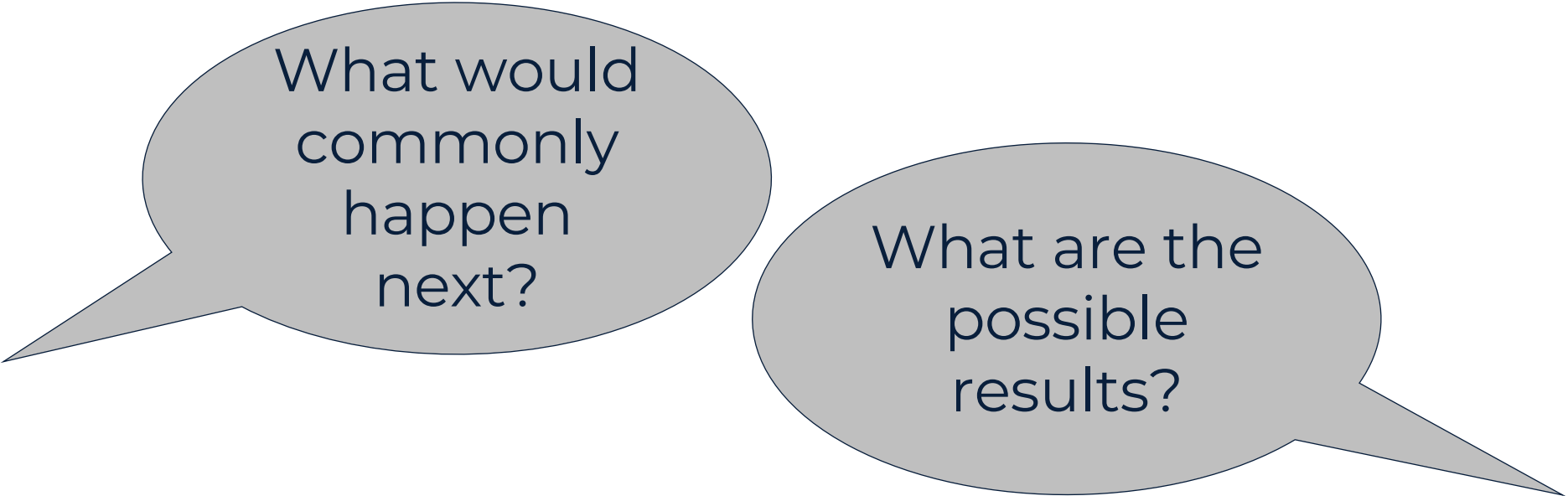
- What to say after Kassie shows why she disagrees with Gabi

Some possible next moves

- Say, 'It is a fraction. Can you make the parts equal?'
- Ask, 'Who can help Antar?'
- Ask, 'Who can say what Antar said?'

What are the considerations? What might be the effect?

- Make clear that his answer is incorrect; clarify the concept for everyone.
- Get other students to participate.
- Encourage students to respond to one another.



What would
commonly
happen
next?

What are the
possible
results?

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

RESULTS

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Kassie, I see what you are saying, but remember that Gabi just showed us what the whole is and that the gray square is one-fourth.

RESULTS

- Kassie’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Kassie, I see what you are saying, but remember that Gabi just showed us what the whole is and that the gray square is one-fourth.
- Kassie, it is really important to identify what the whole is first. Who can show Kassie what the whole is here?”

RESULTS

- Kassie’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.
- Kassie is called out for not knowing what the whole is and she is sidelined.

In the next moment, what is likely to happen?

NORMALIZED NEXT MOVES

- “Kassie, I see what you are saying, but remember that Gabi just showed us what the whole is and that the gray square is one-fourth.
- Kassie, it is really important to identify what the whole is first. Who can show Kassie what the whole is here?”
- “Thumbs up if you agree with Kassie; thumbs down if you disagree.”

RESULTS

- Kassie’s answer is signaled to be incorrect and she is positioned as not having contributed to the work.
- Kassie is called out for not knowing what the whole is and she is sidelined.
- Kassie’s solution is “voted” on by her classmates.

60

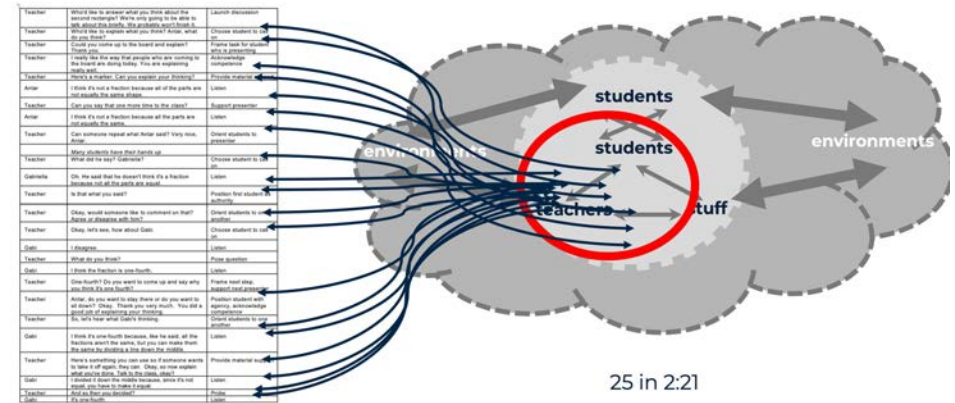


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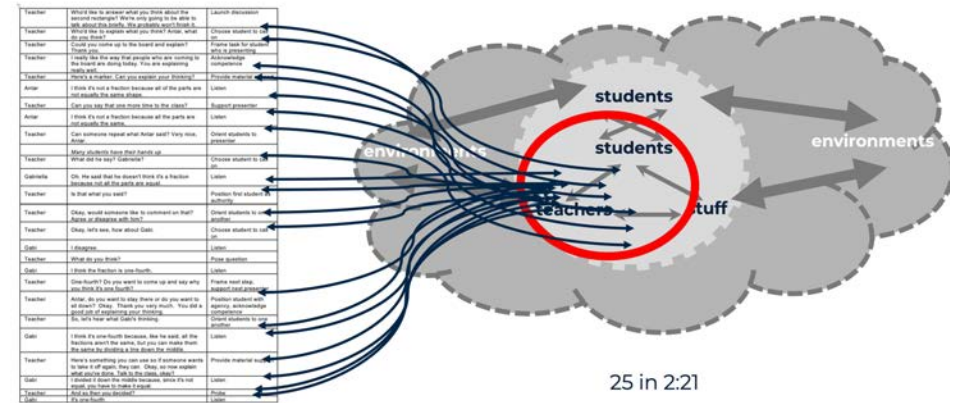
Discretionary spaces are inherent in teaching



- A **discretionary space** is where interpretations, next moves, comments, or questions are necessarily determined by the teacher—and not by a policy or curriculum.
- These interpretations and actions are learned through firsthand experience in society and in school.
- These interpretations and actions are also based on habits.
- (Ball, 2018, in press; Ngo, 2017, Noel, 2018, Sfard)

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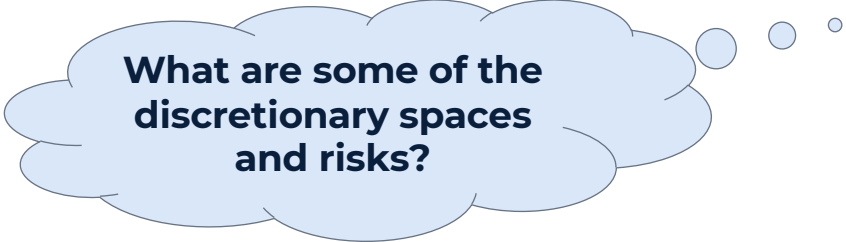
Discretionary spaces are inherent in teaching



The density of discretionary spaces related to advancing children's mathematical flourishing



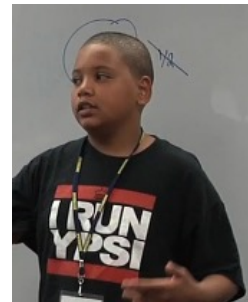
The density of discretionary spaces related to advancing children's mathematical flourishing



**What are some of the
discretionary spaces
and risks?**

The density of discretionary spaces related to advancing children's mathematical flourishing

What are some of the discretionary spaces and risks?



Antar



Gabi



Kassie

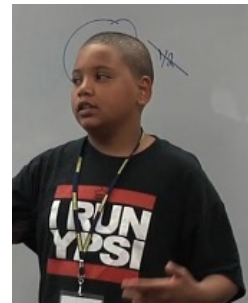


The density of discretionary spaces related to advancing children's mathematical flourishing

- How are these three different African American children—Antar, Gabi, and Kassie—positioned in front of their classmates—as contributing to the mathematics, as lacking understanding? Are their brilliance and humanity seen?
- What is signaled about being a “doer of mathematics”—both what and who?
- What mathematical understanding is developing in the class?
- How are Antar, Gabi, and Kassie experiencing their teacher, their peers, this lesson?
- What are the other children in the class learning about African American children, about who and what it means to be ‘smart’?

(Gholson, 2021; Wilkes, 2021; Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015)

What are some of the discretionary spaces and risks?



Antar



Gabi



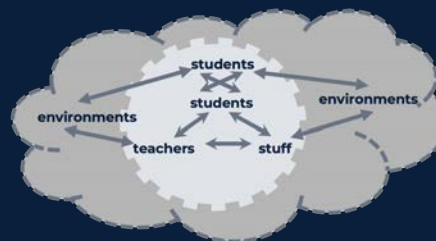
Kassie



In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?



In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?



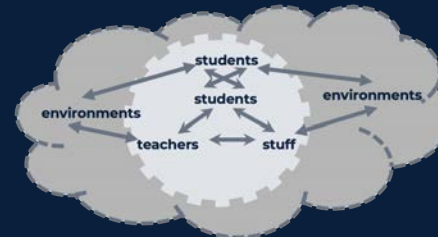
In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?



Seeing Antar's explanation and Kassie's question as key to the class's work

Knowing and using mathematics in teaching (MKT)

Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015



In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?

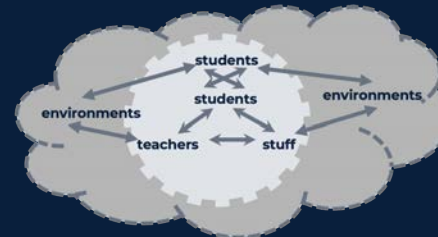


Seeing Antar's explanation and Kassie's question as key to the class's work

Knowing and using mathematics in teaching (MKT)

Seeing what Marquis is doing instead of what one wants to hear

Taking as axiomatic the brilliance of African American children



Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015

In this case, what is some of the work of teaching mathematics for students' mathematical flourishing?



Gholson & Martin, 2014; Langer-Osuna, 2015, 2017; Leonard & Martin, 2013; Martin, 2012, 2015

Seeing Antar's explanation and Kassie's question as key to the class's work

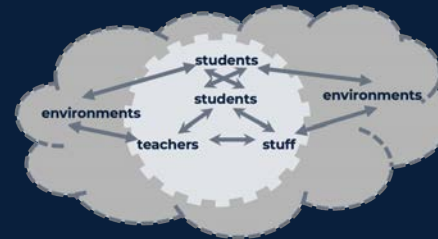
Seeing what Marquis is doing instead of what one wants to hear

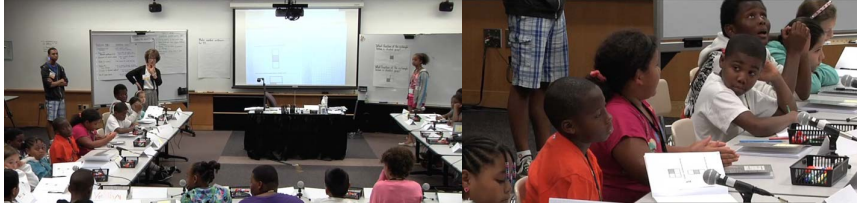
Having things to DO that are not the norm

Knowing and using mathematics in teaching (MKT)

Taking as axiomatic the brilliance of African American children

Having a repertoire of practices that can be adapted and used in contexts

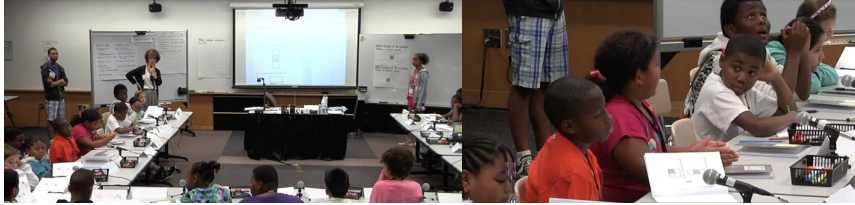




Identifying mathematical learning goals

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

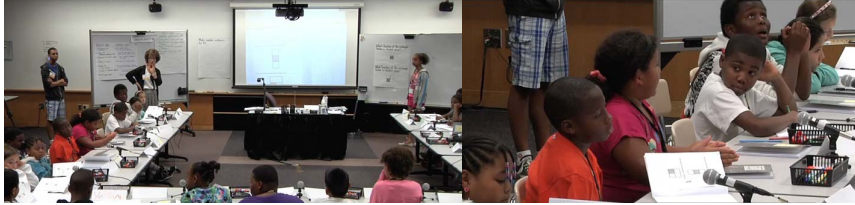




Teaching and learning mathematics

Identifying mathematical learning goals

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.



Teaching and learning mathematics

Identifying mathematical learning goals

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

- Understanding mathematics oriented to others' thinking
- Helping others understand fractions
- Supporting collective mathematical work
- Listening to others' ideas
- Seeing and hearing students
- Supporting mathematical listening, appraising validity, questioning
- Disrupting patterns of what is seen as mathematical competence and who is seen as mathematically competent
- Supporting the development of positive mathematical identities

The argument

We need more research that focuses inside the nuances of actual practice.

1. Fostering young people's mathematical flourishing is essential.
2. Many efforts have been made over decades to do this.
3. Still not a lot has changed inside mathematics classrooms, achievement is flat, and patterns of exclusion and inequity persist.
4. The failure to understand the work of teaching is a foundational flaw.
5. We must probe inside the actual work of teaching mathematics to develop nuanced understanding of the work and its demands. This will contribute to better teacher professional development and to young people's mathematical flourishing.



THANK YOU!



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@deborah_ball



Credits



Image on slide 2:

“Navajo class at Indian Wells Elementary School” by Voice of America

Retrieved from <https://learningenglish.voanews.com/a/navajos-study-to-save-their-language/2999599.html>

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Image on slide 2:

“_JMH3052” by Flickr user US Department of Education

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Image on slide 2:

“The class drawing and writing about the floating pumpkin” by Flickr user lori05871

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Image on slide 2:

"Margaret Cone Head Start Center 3" by Flickr user US Department of Education

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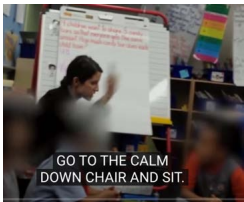


Image on slide 2:

Screenshot of video from "A Momentary Lapse or Abusive Teaching?" The New York Times.

Retrieved from <https://www.nytimes.com/video/nyregion/100000004159212/success-academy-teacher-rip-and-redo-video.html>



Image on slide 2:

Photo from "Cause for Alarm: Addressing North Carolina Early Childhood Suspension and

Expulsion Rates," by Ebonyse Mead, MA, MS, CFLE and Kara Lehnhardt, MBA

Retrieved from <https://www.smartstart.org/cause-for-alarm-addressing-north-carolina-early-childhood-suspension-and-expulsion-rates/>

Credits

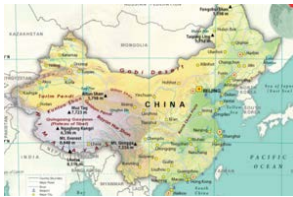


Image on slide 3:
Map of People's Republic of China.
Retrieved from <https://www.mapsofworld.com/answers/geography/what-are-the-key-facts-of-peoples-republic-of-china/#>



Image on slide 3:
Map of United States of America.
Retrieved from: <https://geology.com/world/the-united-states-of-america-satellite-image.shtml>



Image on slide 3:
Map of Turkey.
Retrieved from <https://www.bespoketurkey.com/top-sights-in-turkey/>

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Image on slide 3:
Map of Denmark.
Retrieved from <https://www.worldatlas.com/maps/denmark>



Image on slide 3:
South Africa Provinces Map.
Retrieved from <https://hansjohnnie.blogspot.com/2020/08/south-africa-provinces-map.html?m=0>



Image on slide 3:
Map of Brazil.
Retrieved from <https://www.go-today.com/brazil-vacations.aspx>

Credits



Image on slide 3:

Political Map of India.

Retrieved from <https://surveyofindia.gov.in/pages/political-map-of-india>



Image on slide 13:

“Two groups of people arguing and fighting.”

Retrieved from https://www.freepik.com/premium-vector/two-groups-people-arguing-fighting-conflict-among-people-angry-characters-having-argument-disagreement-vector-illustration-colleagues-having-debate-misunderstanding_21715825.htm



Image on slides 14–15:

Photo from “Professional Development, K20 Center.

Retrieved from <https://k20center.ou.edu/professional-development/>

Credits



Læringsplaner MAT01-05

Curriculum for Mathematics year 1–10

This is a translation from Norwegian Nynorsk of the official Norwegian subject curriculum text.

Established as regulations by the Ministry of Education and Research on 15 November 2019. The examination scheme was established by the Ministry of Education and Research on 29 June 2020.

Valid from 01.08.2020

Image on slides 14–15:

Screenshot of title page of *Curriculum for Mathematics year 1–10* (English version), Norwegian Directorate for Education and Training.

Retrieved from <https://data.udir.no/kl06/v201906/laereplaner-lk20/MAT01-05.pdf?lang=eng>



Image on slides 14–15:

Cover of *Illustrative Mathematics: Grade 7*, 2019.

Retrieved from <https://k12.kendallhunt.com/product/illustrative-mathematics-grade-7-student-edition-set>



Image on slide 16:

Photo from “What do teachers need this school year? Laura McClure from TED-Ed resolved to find out,” by Laura McClure, TED, September 7, 2016.

Retrieved from <https://ideas.ted.com/how-to-help-a-teacher-out/>



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Credits

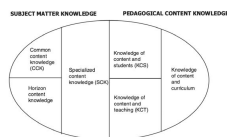


Image on slide 28:

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
<https://doi.org/10.1177/0022487108324554>



Image on slide 28:

Headline from “The Unrealized Promise of High-Quality Instructional Materials” by David Steiner, *State Education Standard*.
Retrieved from <https://www.nasbe.org/the-unrealized-promise-of-high-quality-instructional-materials/>

2. Sandra cut her string chain into equal pieces as shown in the rectangles below. In the blanks below, name the fraction of the string chain represented by the shaded part.

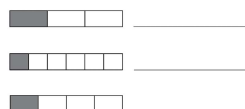


Image on slides 38–40:

Problem from Eureka Math, Grade 3, Module 5, Lesson 1

Retrieved from

https://cdn2.hubspot.net/hubfs/3454910/Florida%20adoption%20materials/EurekaMath_G3M5_UTE_FL.pdf