Comparing Tasks

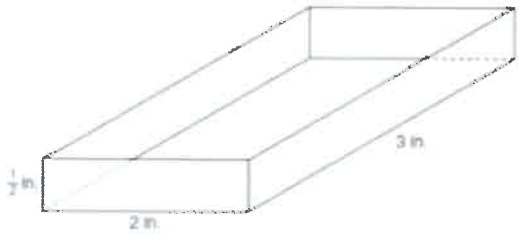
1

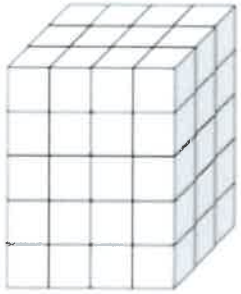
Read these three tasks. Consider:

- What do students have to know and do to solve each task?
- What is similar about the tasks? What is different?

Turn and talk to a neighbor about your observations.

(A) Mr. Muñoz drew a box in the shape of a rectangular prism, as shown below. What is the volume of the box?	(B) Which expression(s) below can you use to determine the volume? Explain why that works. 20 x 12 15 x 3 12 x 5	(C) An 80-gallon bathtub is being filled at a rate of 15 gallons every 2 minutes. At this rate, how many minutes will it take to fill this bathtub ¾ full?





4a

Rectangular Prism Task: The Launch

The teacher first introduced students to the notion of a rectangular prism by showing them a prism made of connected cubes, the edges of which were 2cm.

The students then viewed a two-minute video about aquarium tanks in which the narrator talked about how aquarium tanks hold different amounts of water, and how the amount of water is measured in cubic units.

The teacher next showed her students PowerPoint slides of rectangular prisms. For each, she asked her students: "How could we figure out the volume of this shape?" Students suggested several different strategies and began listing operations in response to one of her questions ("Well, what could we do, addition, subtraction..."). Eventually one student mentioned multiplication, at which point the teacher said "Yes!" She then presented students with the formula for finding the volume of a rectangular prism, showing it to them on a power point slide. One student said, "Oh it's like area," to which the teacher replied, "umm hmm [yes]"

The teacher then gave each group of students the same rectangular prism made of connected 2cm cubes and 1 x 1 cm grid paper. She informed them that they were going to work in small groups (of 4-5 students) to figure out the volume of the rectangular prism.

4b

Rectangular Prism Task: Students' Work

NUMBER OF GROUPS: 6

NUMBER OF DIFFERENT SOLUTIONS: 4

***The length of each edge of a cube is 2 cm.**

Strategy 1: *Students first found the area of one side of a single cube. They then multiplied by the number of cubes in a "layer".* One cube side = 4 cm x 4 cm = 16 sq. cm 16 sq. cm x 15 cubes in a layer = 240 sq. cm 240 sq. cm x 4 layers = 960 sq. cm (2 groups)	Strategy 2: *Students found the surface area of each side of the rectangular prism and then started to multiply these areas together. Students seemed to realize the result was too large, but never moved beyond this strategy.* • Surface area of top (and bottom) = 20 cm x 12 cm = 240 sq. cm • Surface area of bigger sides = 20 cm x 16 cm = 320 sq. cm • Surface area of smaller sides = 12 cm x 16 cm = 192 sq. cm 240 sq. cm x 320 sq. cm = 76800 (*seems too much...)* (2 groups)
Strategy 3: *Students tried to figure out the number of the cubes in the "center" of the rectangular prism. Students then figured out the area of a side of a single cube and multiplied by the number of cubes in the center.* One cube side = 4 cm x 4 cm = 16 sq. cm 16 sq. cm x 6 cubes in center of the rectangular prism = 96 sq. cm (1 group)	Strategy 4: *Students measured the length of the edges of the rectangular prism using the grid paper but measured one of the lengths incorrectly. They then multiplied the resulting measures.* Length = 21 cm (measured incorrectly) Width = 12 cm Height = 16 cm 16 cm x 12 cm x 21 cm = 4032 cm (1 group)

4C

Rectangular Prism Task: Whole Class Discussion

The teacher started the discussion by calling on one group, who shared an incorrect solution. The teacher then asked the group questions such as "What numbers did you multiply?" and "Did you add or did you multiply?" Despite these prompts, the students did not arrive at the correct answer.

The next group the teacher called on also shared an incorrect solution. In response, the teacher walked to the students' table and picked up their rectangular prism. She then asked questions directly to the members of this group while the rest of the class listened or chatted quietly. She then interacted with the third group she called on in a similar way after they also shared an incorrect solution.

5

Identifying Productive Instructional Improvement Goals for Teachers

Students' solution strategies

- Were all students able to work meaningfully on the task(s)?
- Was the range of student strategies rich enough to have a productive discussion?

No

Classroom Culture and Routines

- Classroom management
- Timing/pacing of lesson

Instructional Resources

- Task(s)
- Tools and manipulatives

Earlier Phases of a Lesson

- Launch
- Small group and/or individual work

Reasoning behind Instruction

- Mathematical knowledge for teaching
- View of students' current mathematical capabilities
- Vision of high-quality mathematics instruction
- Planning process

Yes

Classroom Culture and Routines

- Comfort in sharing thinking publicly
- Turn-taking routines

Five Practices for Orchestrating Productive Mathematics Discussions

- *Anticipating* what strategies students will use in solving a problem
- *Monitoring* students' work as they approach the problem in class
- *Selecting* students whose strategies are worth discussing in class
- *Sequencing* those students' presentations to maximize their potential to increase student learning
- *Connecting* the strategies and ideas in a way that helps students understand the mathematics learned

2a

Elementary Task: Five Monkeys in Two Trees

Students: Grade 7 (12-13 year olds)
Long-term learning goal: Investigate and compare linear relationships (rate of change, initial value), using equations, tables, and graphs as tools

With 1-2 colleagues

(1) **Anticipate** at least 2-3 ways students might approach/ solve the task, given the long-term learning goal.
(2) **Discuss**: What would students need to understand in order to begin to work meaningfully on this task?

What are all the different ways that five monkeys could be in the two trees?

How do you know that you found all of the ways?

2b

Secondary Task: Dollars for Dancing

Adapted from *Connected Mathematics Project 2* (Lappan et al., 2009)

Students: Grade 7 (12-13 year olds)
Long-term learning goal: Investigate and compare linear relationships (rate of change, initial value), using equations, tables, and graphs as tools

With 1-2 colleagues

(1) **Anticipate** at least 2-3 ways students might approach/ solve the task, given the long-term learning goal.
(2) **Discuss**: What would students need to understand in order to begin to work meaningfully on this task?

Three students at a school are raising dollars for the school's Valentines Dance. All three decide to raise their money by having a dance marathon in the cafeteria the week before the real dance. They will collect pledges for the number of hours that they dance, and then they will give the money to the student council to get a good DJ for the Valentines Dance.

- Rosalba's plan is to ask teachers to pledge $3 per hour that she dances.
- Nathan's plan is to ask teachers to give $5 plus $1 for every hour he dances.
- James's plan is to ask teachers to give $8 plus $0.50 for every hour he dances.

Part A. Create at least three different ways to show how to compare the amounts of money that the students can earn from their plans if they each get one teacher to pledge.

Part B. Explain how the hourly pledge amount is represented in each of your ways from Part A.

Part C. For each of your ways in Part A explain how the fixed amount in Nathan's plan and in James's plans is represented.

Part D. For each of the ways in Part A show how you could find the amount of money collected by each student if they could dance for 24 hours.

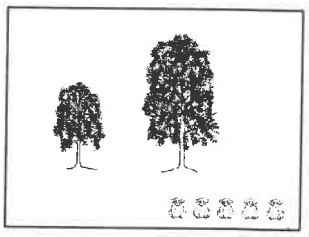
Part E. Who has the best plan? Justify your answer.

3a

Launch – Elementary Task – Five Monkeys, Two Trees Task

All of the first-grade students sit on the carpet in front of the teacher.

The teacher projects an image of two trees and five monkeys on the screen.



Teacher: I'd like everyone to look at this picture. Let's first think quietly to ourselves, what do you notice? *(waits 10 seconds)* Turn and talk to your partner and share, what do you notice? Make sure that each person shares two things they notice about the picture.

Students talk to their partners for about 30 seconds. Teacher circulates to hear what students are saying. All students notice the trees and the monkeys. Some students have counted the trees and monkeys, some have not. Some students have noticed that one tree is small, and the other tree is big.

Teacher: Let's come back together and share what we noticed. You might share what *you* noticed, or what *your partner* noticed. If you saw the same thing as another student, you can show your "me, too" sign. (Students have been taught to use the American Sign Language sign to indicate "me, too" or "I agree"). What did you notice, Max?

Max: I noticed there are monkeys. (All of the students indicate "me, too.")

Teacher: Everyone noticed there were monkeys! How many monkeys are there? Charlie?

Charlie: There are five monkeys! (Many, but not all, students indicate "me, too.")

Teacher: Charlie said there are *five monkeys*. Should we count them together to check?

The teacher points to the monkeys, one by one, as students count together: One, two, three, four, five.

Teacher: OK, so we noticed there are 5 monkeys. What else did you notice? Lucy?

Lucy: We noticed there are trees, one is little and one is big.

Teacher: Jenna, what did Lucy notice?

Jenna: Lucy said there is one little tree and one big tree.

Teacher: Hmm. So how many trees are there altogether?
Students: Two!

Teacher: Wonderful work sharing what you all noticed. So there are 5 monkeys (points to each of the 5 monkeys) and 2 trees (points to the 2 trees). Do you know this about monkeys? Monkeys love to play in trees! Here is what we're going to think about together today. We're going to figure out ALL the *different* ways that these FIVE monkeys could be in these TWO trees!

Teacher: Quietly, in your mind, picture for yourself, what's ONE way that the five monkeys could be in these two trees?

(Teacher waits 15 seconds)

Teacher: Maleda, what are you picturing in your mind? What's ONE way that the five monkeys could be in these two trees?

Maleda: One is in the small tree and two in the big tree.

Teacher: So Maleda saw one of the monkeys in the small tree (points to the leftmost monkey and then moves her finger to the small tree) and two monkeys in the big tree (points to the next two monkeys and then moves her finger to the big tree). Does anyone have a comment for Maleda?

(Several students raise their hands.)

Teacher: Tanya?

Tanya: But there are two more monkeys. They are *all* playing in the trees. (Several students indicate with their hands, "I agree.")

Teacher: Tommy, what did Tanya say?

Tommy: Tanya said that all of the monkeys are playing in the trees, so those two monkeys (points to the rightmost two monkeys) need to be in the trees, too!

Teacher: Yes, ALL of the monkeys (circles all of the monkeys with her finger) are playing in the trees. None will be on the ground. Maleda, what are you picturing now?

Maleda: Those monkeys (points to the rightmost 2 monkeys) are in the trees, too.

Teacher: OK, so everyone should be picturing different ways that ALL of the monkeys could be in these two trees. None will be on the ground. Now, I am going to ask you to work in partners to figure out ALL the different ways that these 5 monkeys (points to all of the monkeys) could be in these two trees (points to the trees)! I'm going to give each pair of partners a blank piece of paper. I'd like you to record ALL the different ways on this paper. I'll be asking you to *show me* the different ways!

(Teacher circulates. She pays attention to the combinations students generate (e.g., do they treat 0 monkeys in the small tree and 5 monkeys in the big tree as different from 5 monkeys in the small tree and 0 monkeys in the big tree), and how they represent their combinations. As students finish, she will press: How do you know that you found all of the ways?)

36

Launch – Secondary Task – Dollars for Dancing Task

All of the seventh grade students are seated at their desks, in groups of 3-4.

Teacher: We are ready to begin our lesson for today. And this is what the lesson is going to be about. I want you to take a look at the pictures here ... and see if you can tell anything that might be going on in today's math class that we're going to be talking about.

The teacher projects images of dance marathons.

Luna: Dance!

Teacher: OK. Dance. Somebody add on to that please. James.

James: A dance marathon.

Teacher: A dance marathon. Let's see if anybody can add on to what a dance marathon is or talk about it. Marcus?

Marcus: A group of people.

Teacher: Okay. A dance marathon might be a group of people. Let's talk about what happens at a dance marathon or why people might do it. Okay, Luis?

Luis: A certain amount of people dance for like a certain amount of time.

Teacher: Okay. People dance for a certain amount of time. Can anybody add on to that? What else do you know about a dance marathon?

James: Uh...it's usually held for a long time. A long period of time.

Teacher: Okay. Alright. A marathon is usually held for a long period of time. Okay, so they've got to dance for a long time. Anybody else know *why* you would have a dance marathon?

Students share several reasons: "To show people you can dance," "To see who lasts longest," "Tradition," "Raise Money."

Teacher: Okay to raise money. Stop right there. Today, our dance marathon that we talk about is going to be about raising money. This one here (pointing at image on the screen). This dance marathon, this dance marathon (continuing to point at the images on the screen). All of those are dance marathons that are held

in universities. That's where those are from, and they're to raise money. Why might people have a dance marathon to raise money?

Students share: "Donate to charities," "Help people," "Help schools," "Help to pay for college."

Teacher: Now think of [our school]. What's going on in [our school] in about a week and a couple of days that we might want to raise money for? William?

William: The Valentine's Dance.

Teacher: Okay. A Valentine's dance. Right. And there was trouble raising money. We had something called a penny war because we didn't have enough money to pay for what at the dance? It was something really important that we need at the dance. Maria?

Maria: The music!

Teacher: The music. Right. We need money for the DJ. Here we go. This is the math that we're going to do. Our math is going to be about raising money for the DJ that's coming up at your dance.

There's two ways you can raise money in a dance marathon that we're going to talk about. One way is to dance for a long time. James and some others said a marathon takes a long time and people dance for a long time. So if you dance for a long time, and let's say I give you 50 cents every hour you're going to make a lot of money. But there's *another* way that you could raise money and that is to ask for a *pledge*. Not per hour, but just a *donation*. Okay we call that a *donation*. And you might go up to your teacher and say, can you give me $6 for being in the dance marathon. Now that's different. Can anybody explain why how that is different if I say can you give me $6 or instead can you give me 50 cents an hour. What's the difference, Shawn?

Shawn: You start with some money and then they add more money, if they can.

Teacher: Lisa, add onto that.

Lisa: It's either they pay you up front or you continue so they continue to pay you for however long you dance.

Teacher: Great. So we have one where they pay you up front, one where they add on to it. How many people understand kinda what this is...what we're talking about here? (Waits to see students' hands). Two kinds of fund raising. Pay you up front or pay you where you add on. Janelle, can you say it in your words? There's two ways that you could raise money, what are they?

Janelle: Well like one of them you already start with it and the other one you have to kind of work for it to get more.

Teacher: Exactly. I like the way that's worded. One of them you start with it, you just have it. The other one you got to work for it to get the money. That's your work today. You're going to create tables, equations, and graphs where people either get the money at the start or they have to work for it.

(Teacher passes out the Dollars for Dancing Task to students. Students work in groups of 3-4 to solve the task.)