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Zooming in and zooming out: the color chart of digital technology in mathematics education

Paul Drijvers

Copenhagen, 2023-03-17

www.matematikdidaktik.dk



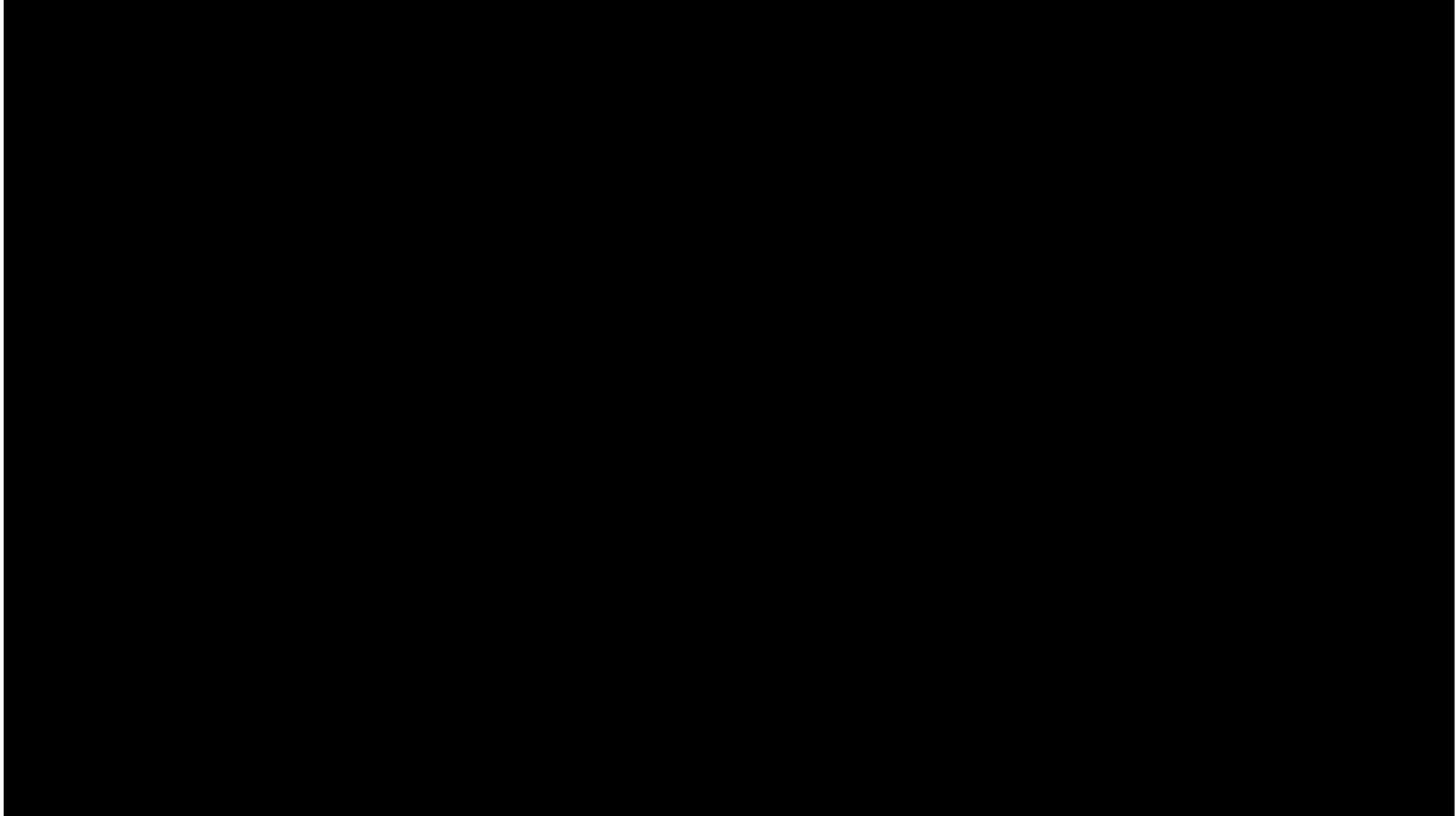
**Paul Klee (1930):
"Couleur Table Qu 1"
Kunstmuseum Basel**



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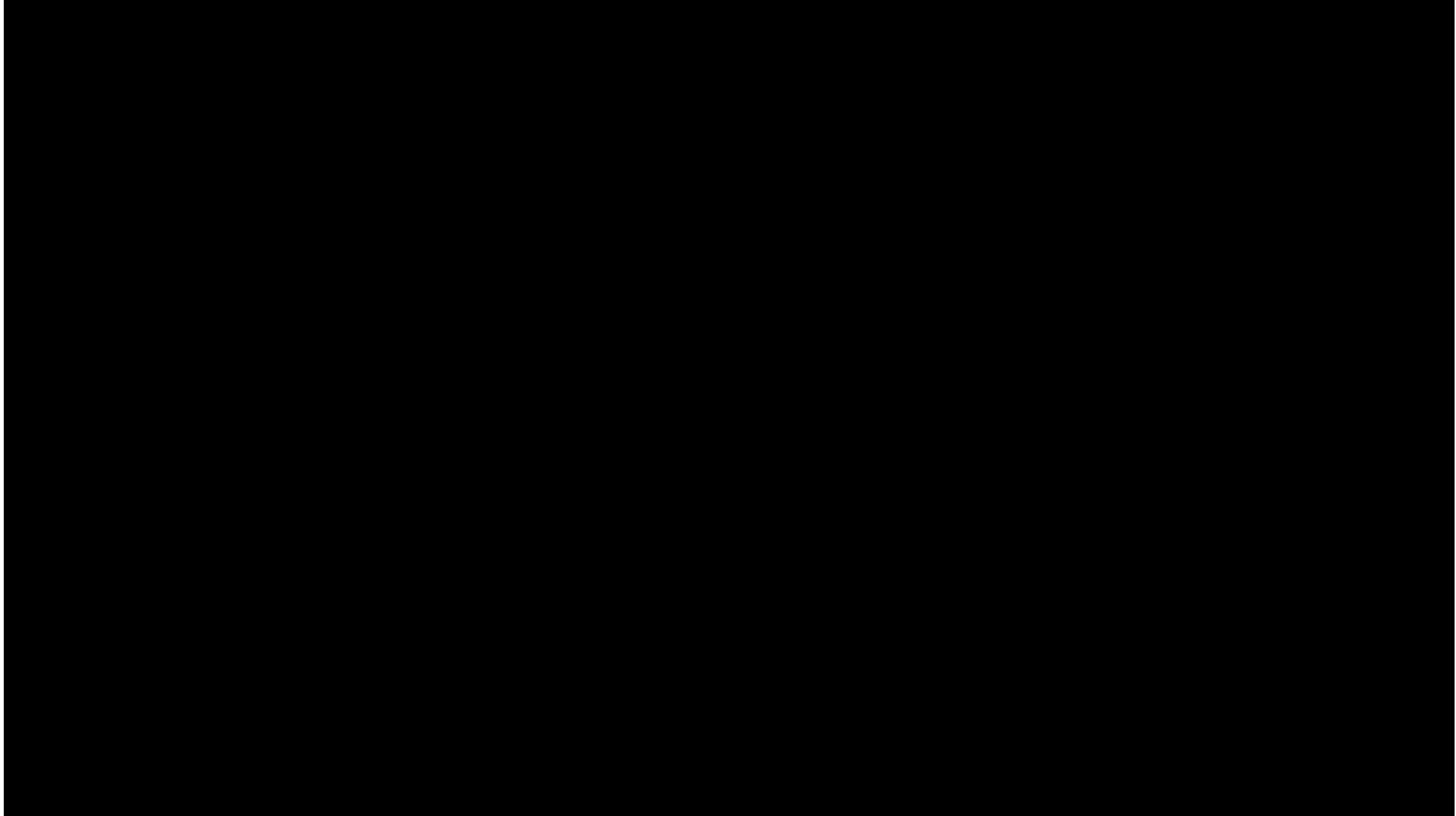
	Tools	Teaching	Theory
Past			
Present			
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Early graphing tools example: Local linearity

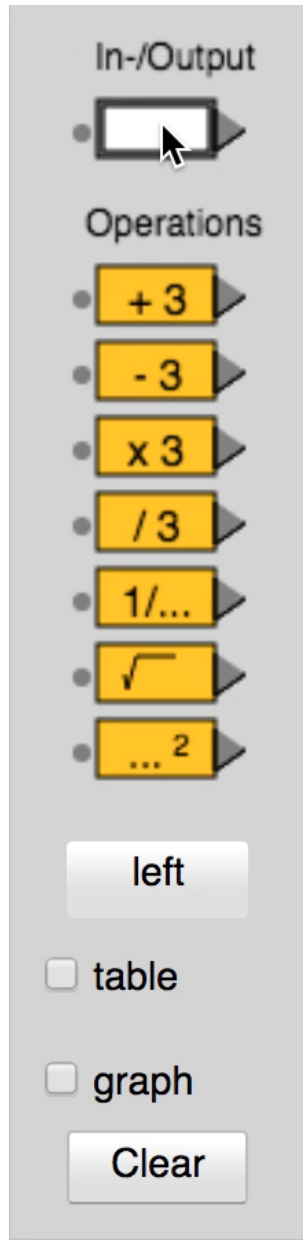




Early Dynamic Geometry example: Perpendicular bisectors



Early applets example: Arrow chains



The screenshot shows a vertical panel with the following elements from top to bottom:

- In-/Output**: A text label above a white rectangular input field with a mouse cursor pointing to it.
- Operations**: A text label above a vertical list of seven yellow buttons, each with a black arrow on its right side:
 - $+ 3$
 - $- 3$
 - $\times 3$
 - $/ 3$
 - $1/\dots$
 - $\sqrt{\quad}$
 - \dots^2
- left**: A white button with a grey gradient.
- table**: A checkbox followed by the text "table".
- graph**: A checkbox followed by the text "graph".
- Clear**: A white button with a grey gradient.

Solve($\sin(x) = \cos(x)$, x)

$$\rightarrow \left\{ x = k_1 \pi + \frac{1}{4} \pi \right\}$$

Expand($(x + y)^{10}$)

$$\rightarrow x^{10} + 10 x^9 y + 45 x^8 y^2 + 120 x^7 y^3 + 210 x^6 y^4 + 252 x^5 y^5 + 210 x^4 y^6 + 120 x^3 y^7 + 45 x^2 y^8 + 10 x y^9 + y^{10}$$

Factor($100!$)

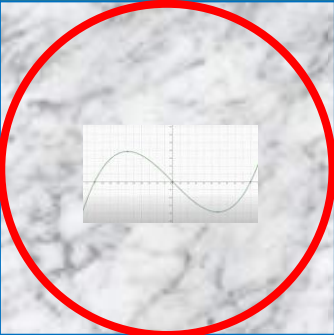
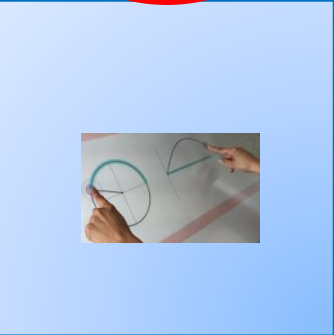
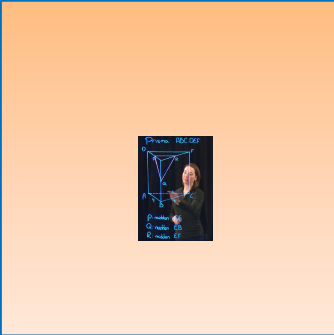

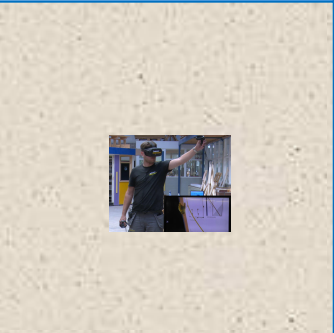
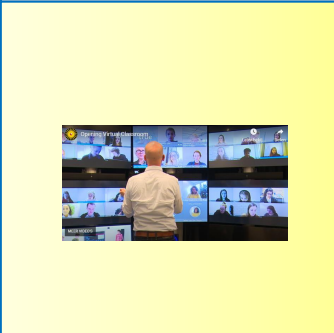
$$\rightarrow 2^{97} \cdot 3^{48} \cdot 5^{24} \cdot 7^{16} \cdot 11^9 \cdot 13^7 \cdot 17^5 \cdot 19^5 \cdot 23^4 \cdot 29^3 \cdot 31^3 \cdot 37^2 \cdot 41^2 \cdot 43^2 \cdot 47^2 \cdot 53 \cdot 59 \cdot 71 \cdot 73 \cdot 79$$

$$\int x e^x dx$$

$$\rightarrow e^x (x - 1) + c_1$$



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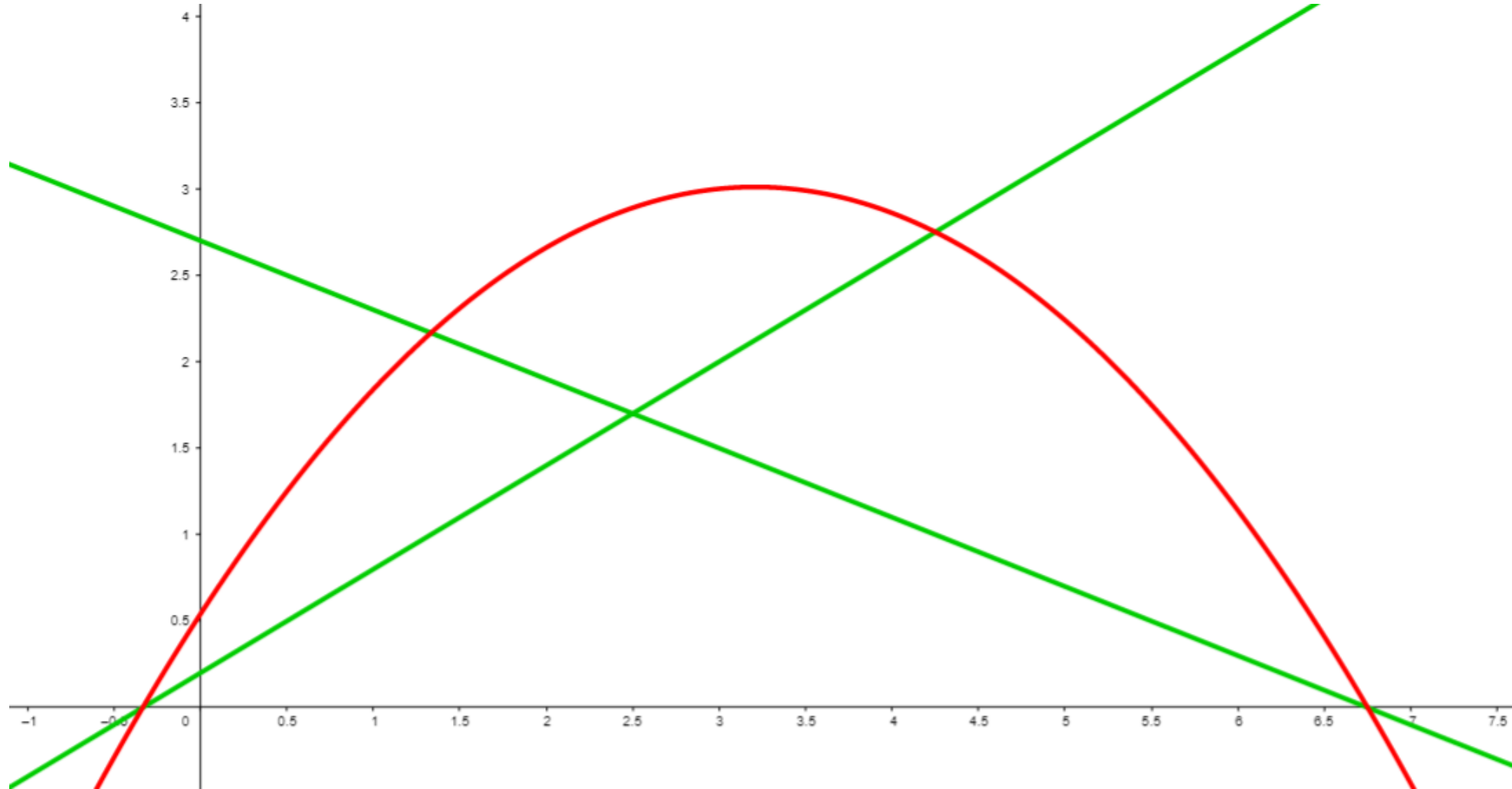
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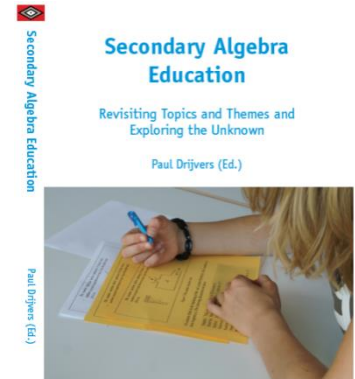
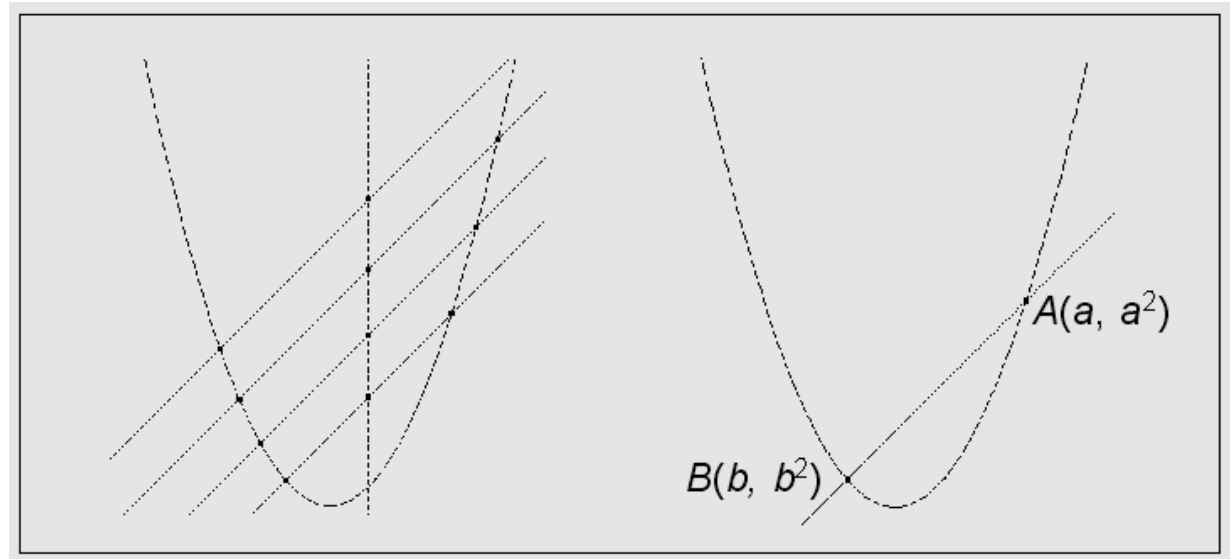
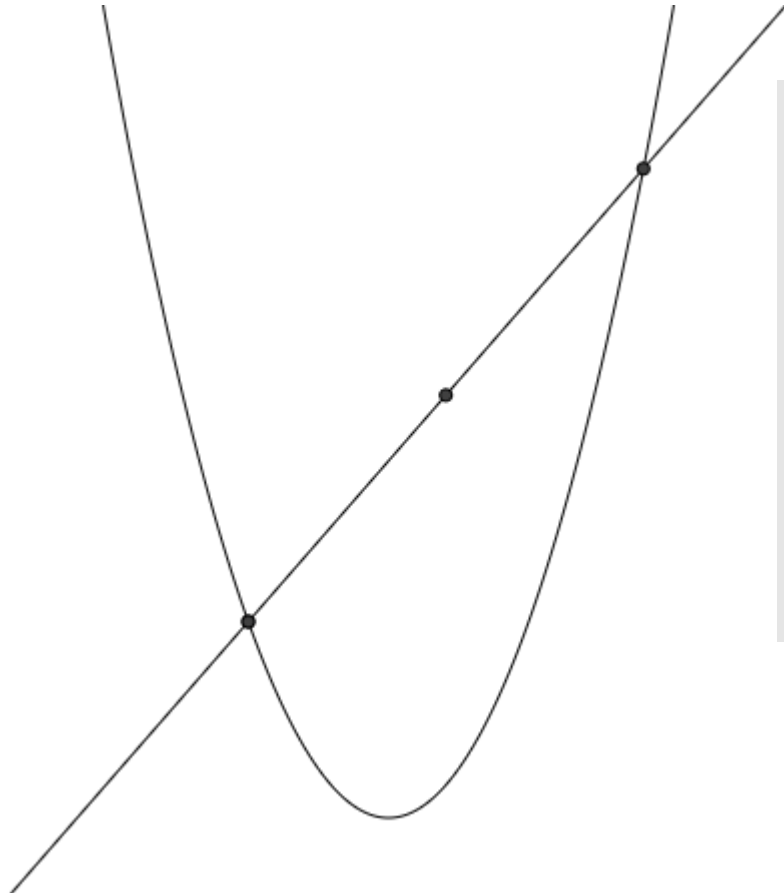
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Early Teaching Ideas: Exploration

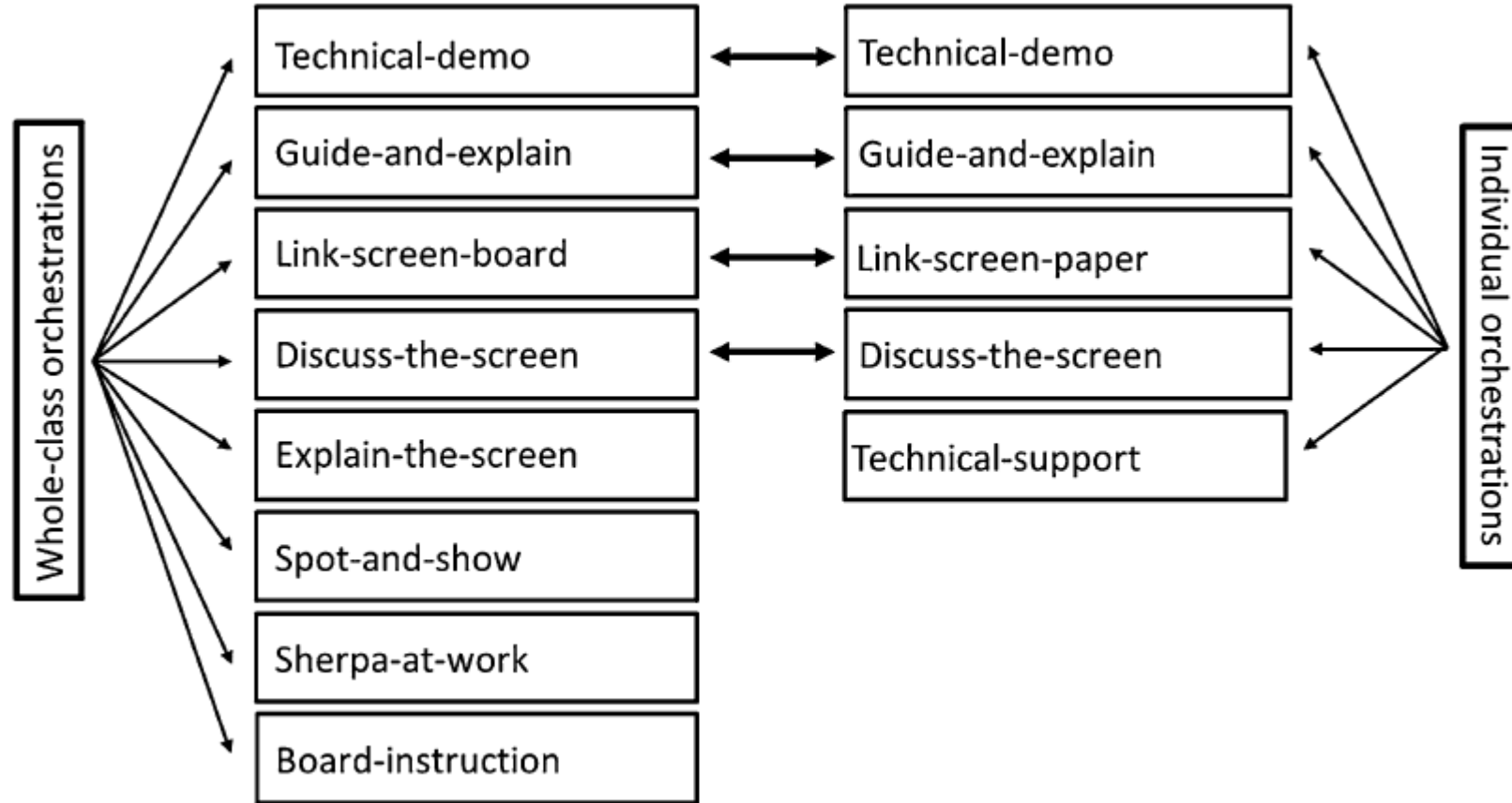


Early Teaching Ideas: From Exploration to Proof

show



Early Teaching Ideas: Orchestrations





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	Tools	Teaching	Theory
Past			
Present			
Future			<p>THE FUTURE IS HERE: What we know - 20% will be useful and lead to learning gains ... but human core teaching elements ... will be important for the delivery of student-led instruction</p> <p>What we should find out: - How does VR impact on such scenarios and learning gains? - What are the skills in the digital era? - Can we really do 'competence-based' edU? - What are theoretical principles for learning and teaching in the digital era? - How to make up 'equipment' (classroom research building)?</p>



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Early Theory Development: Instrumental Genesis

Indicate the unknown to solve

An equation should contain an = sign

Notice the scope of the square root sign

$\text{solve}(x^2 + b \cdot x + 1 = 0, x)$

$x = \frac{\sqrt{b^2 - 4} - b}{2}$ or $x = \frac{-\left(\sqrt{b^2 - 4} + b\right)}{2}$

'solve with respect to x ' = 'express x in terms of b '

A solution can be an expression

Published: October 2002

Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work

Michèle Artigue

International Journal of Computers for Mathematical Learning 7, Article number: 245 (2002) | [Cite this article](#)

1716 Accesses | 344 Citations | [Metrics](#)

Early Theory Development: Instrumental Orchestration



 Springer Link

Published: [September 2004](#)

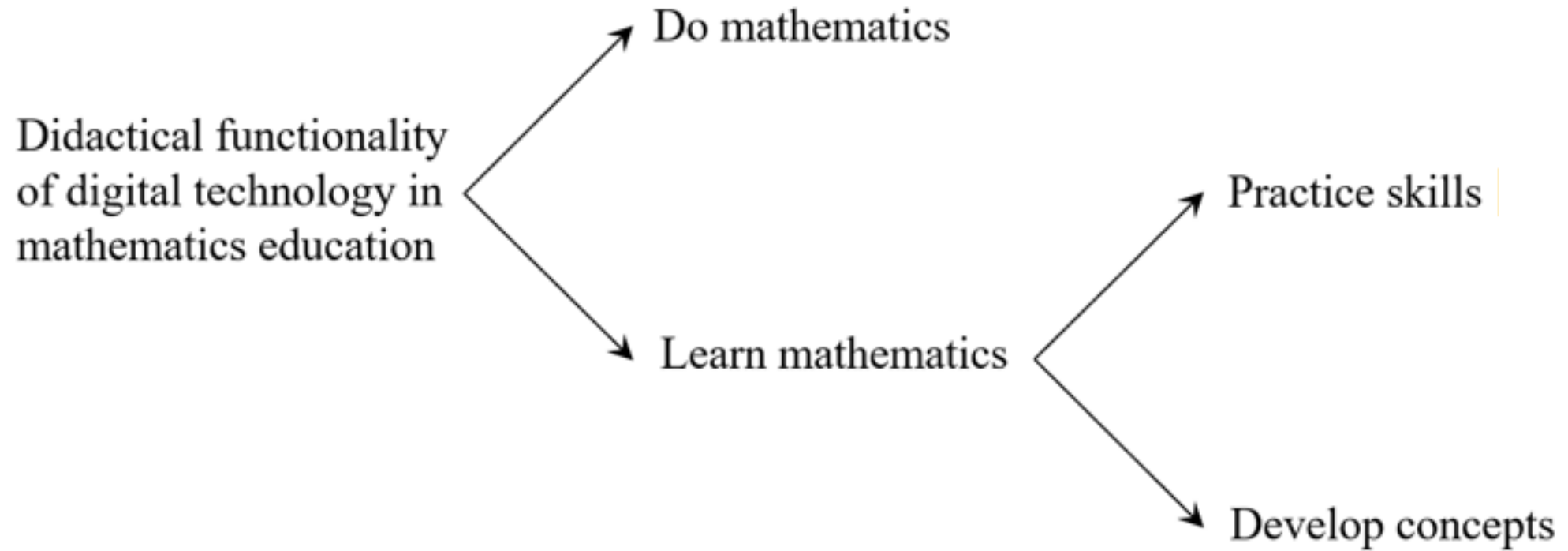
Managing the Complexity of Human/Machine Interactions in Computerized Learning Environments: Guiding Students' Command Process through Instrumental Orchestrations

[Luc Trouche](#) 

[International Journal of Computers for Mathematical Learning](#) **9**, Article number: 281 (2004) | [Cite this article](#)

1221 Accesses | **219** Citations | **3** Altmetric | [Metrics](#)

Early Theory Development: Does it work?



THE PAST IN SHORT:

- Spectacular mathematical tool development
- Focus on exploration and concept development
- Attention to subtlety of student-tool interaction and to teacher orchestrations
- Positive effects, though not spectacular



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***What* technology do you use in your teaching?**



What do you use technology *for* in your teaching?



Integration of tools

 Perspectives

 Graphing

 CAS

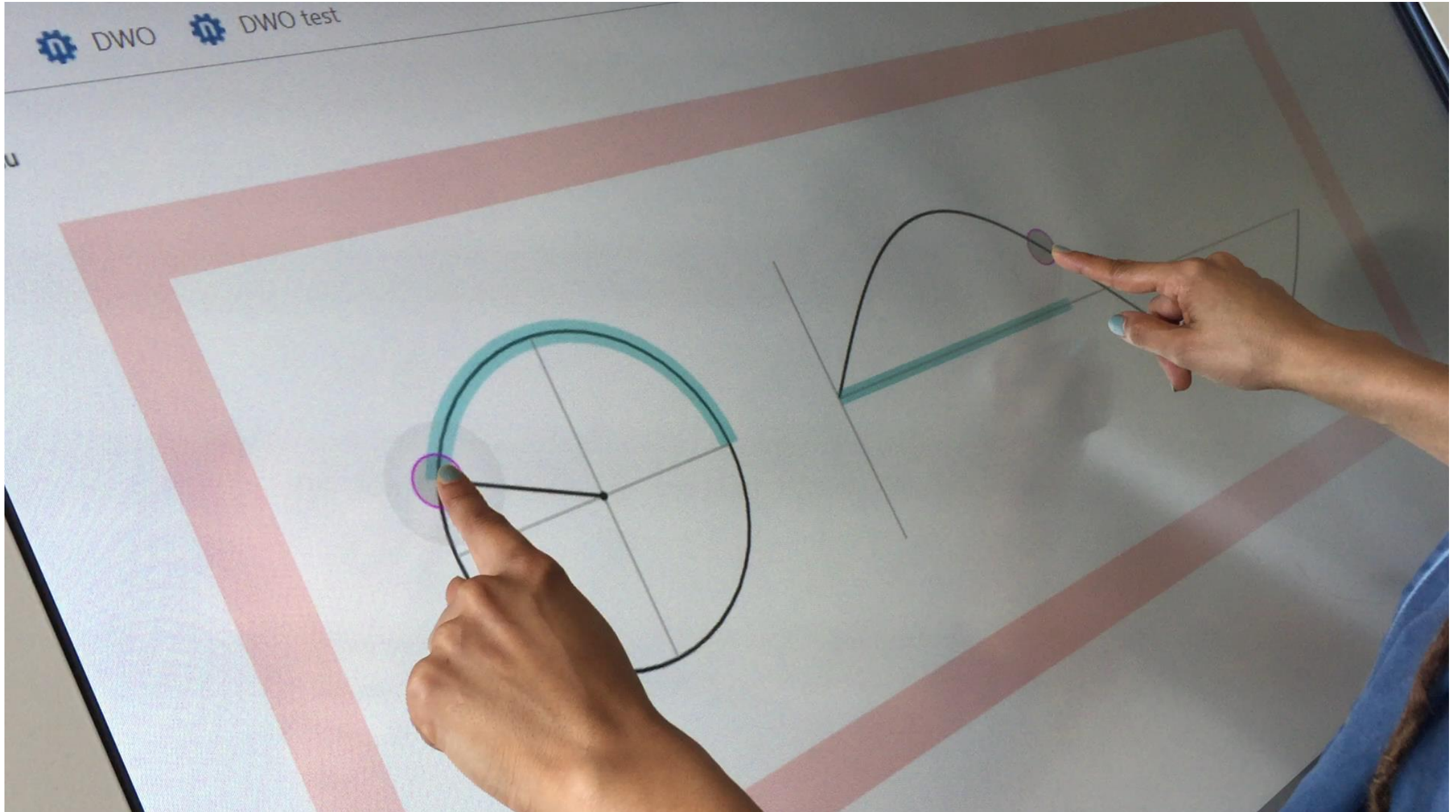
 Geometry

 3D Graphics

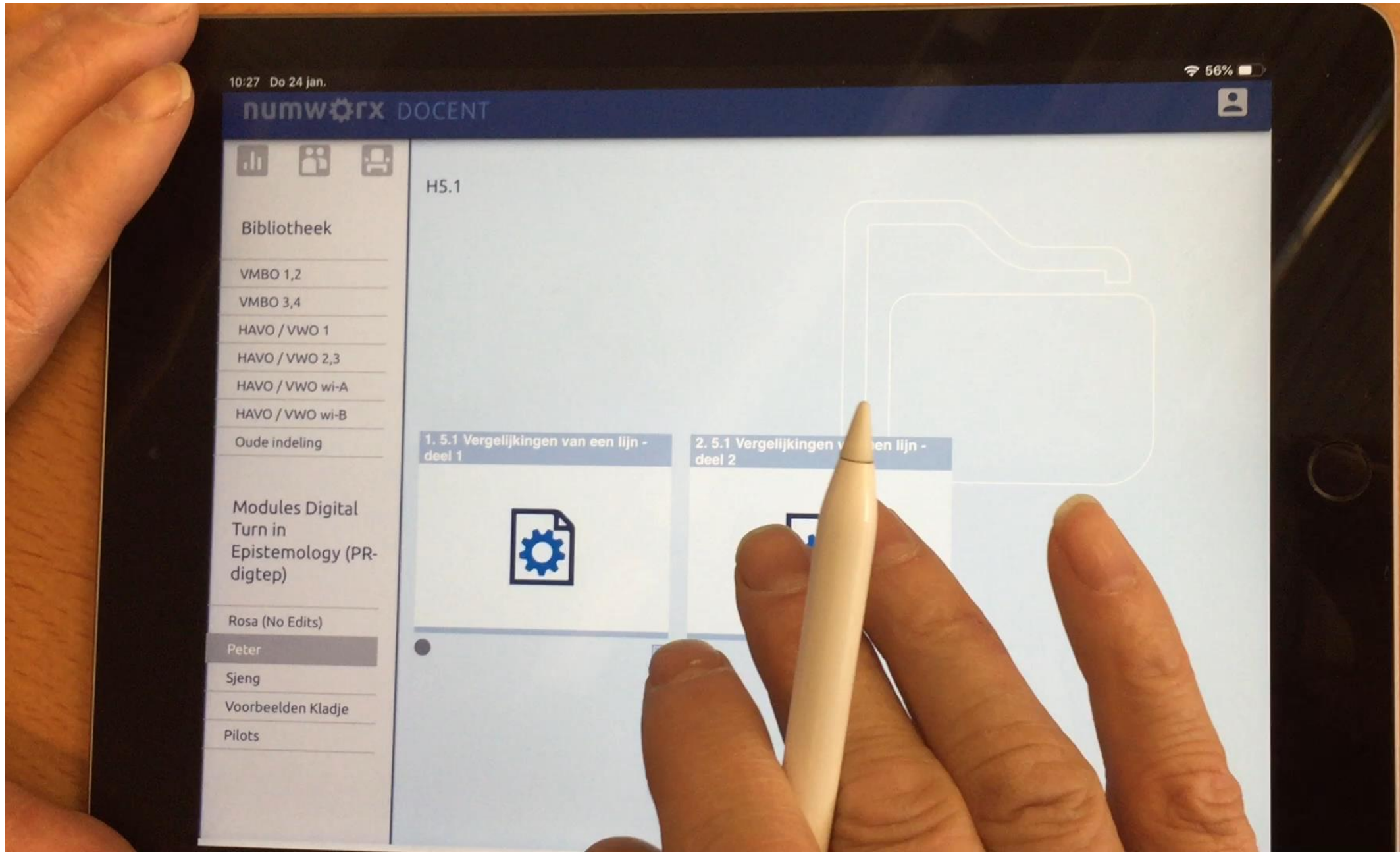
 Spreadsheet

 Probability

New technologies (1)



New technologies (2)



New technologies (3)





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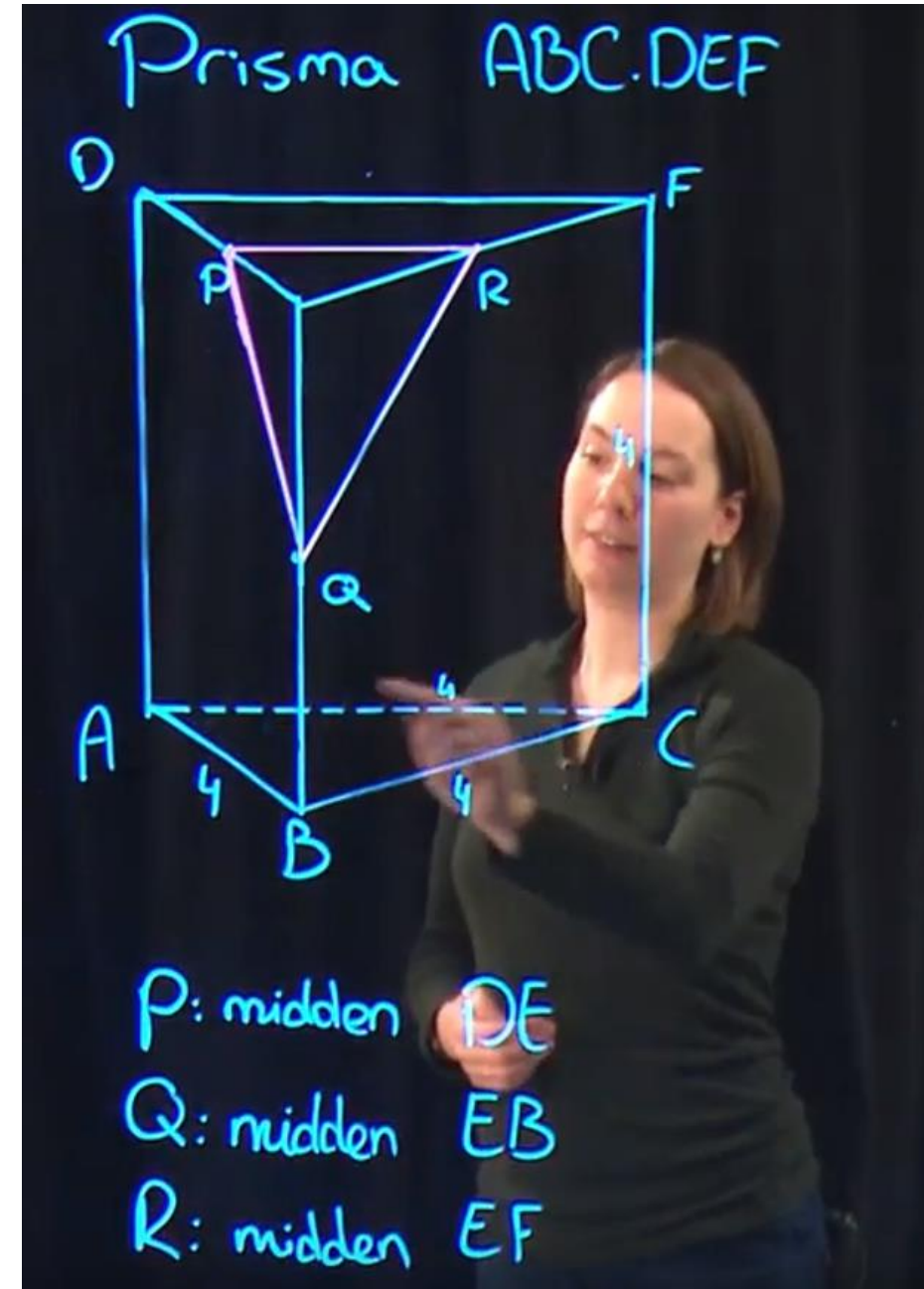
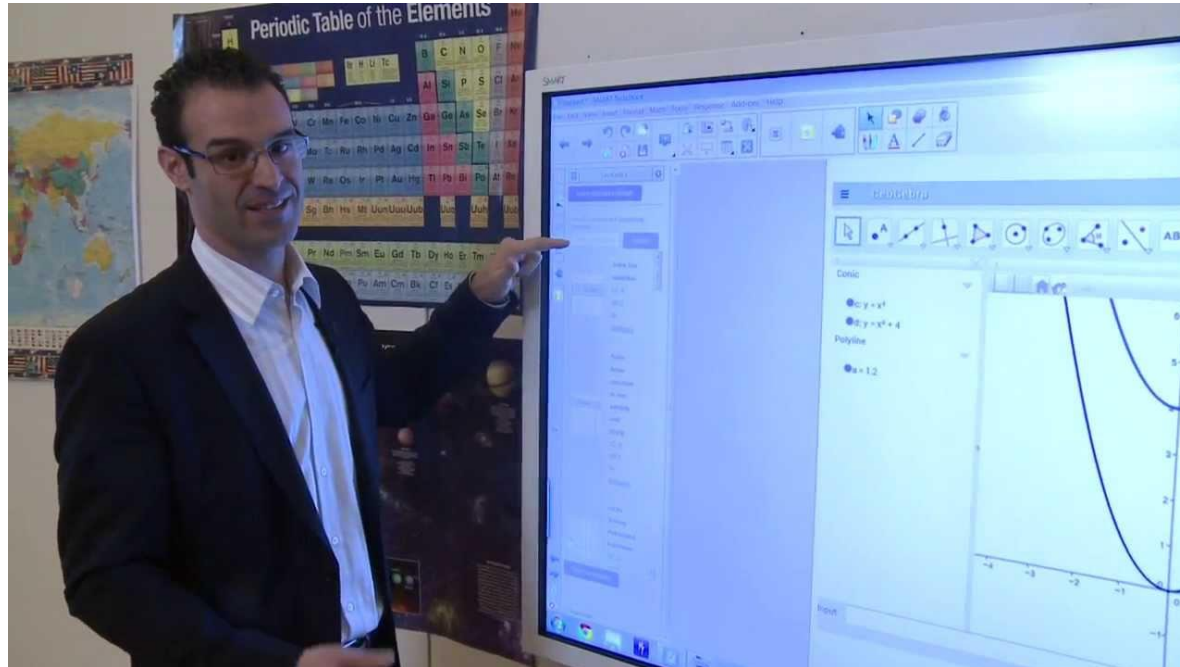
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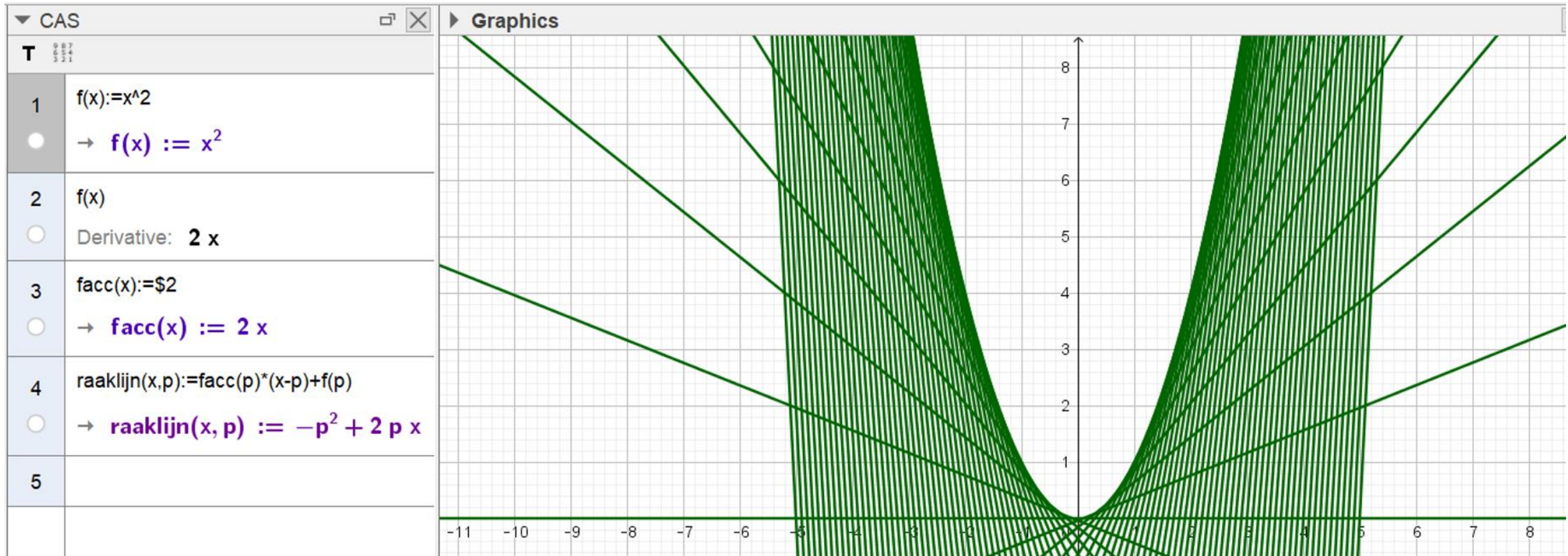
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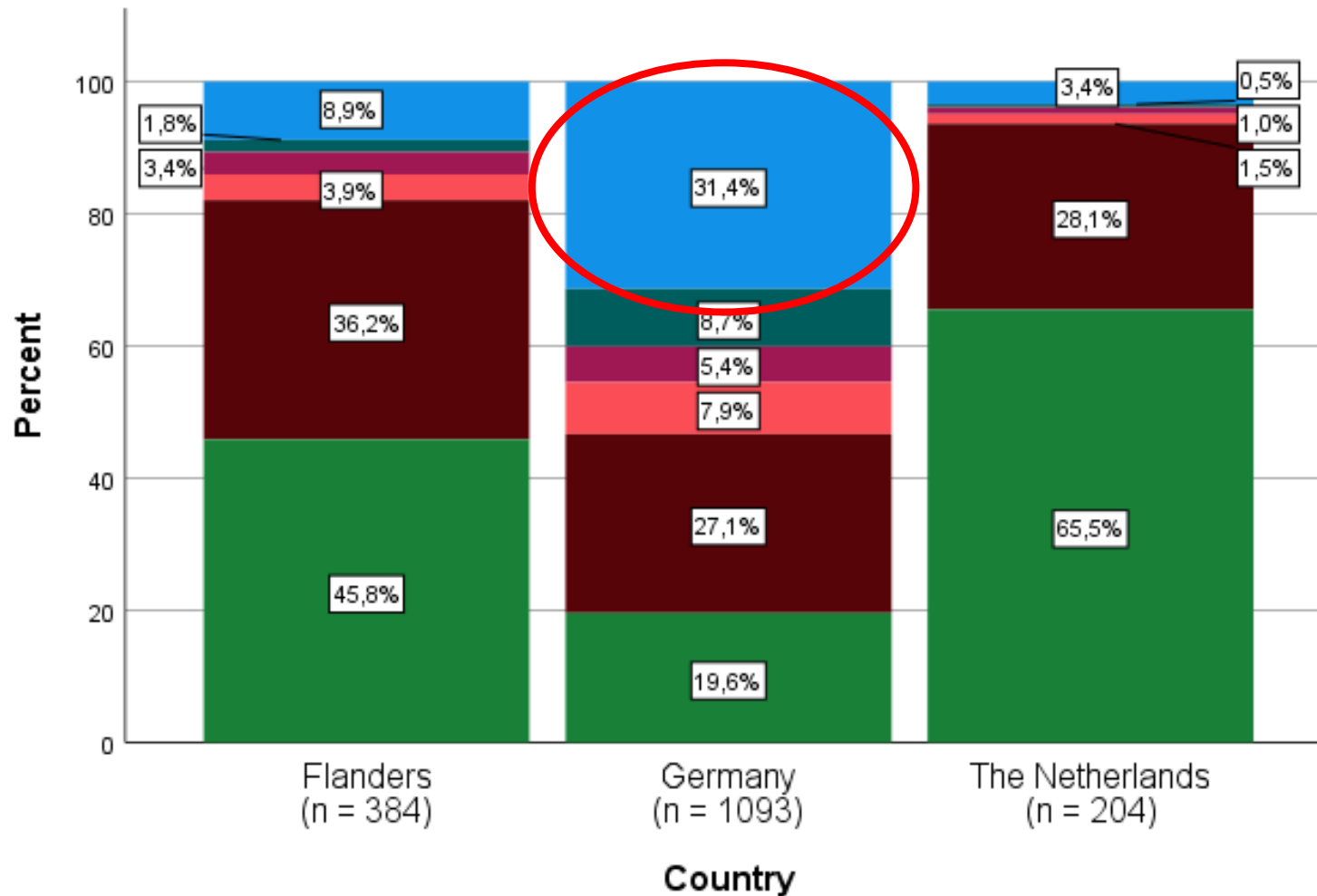
Present teaching practices: advanced settings



Present teaching practices: new topics / approaches



Present teaching practices: ERT experiences through COVID



Lehrer

Unterrichte lieber ungewöhnlich

Deutsche Mathelehrer tun sich schwer mit digitalen Schulstunden aus der Ferne.

Von **Thomas Kerstan**

29. Juli 2020, 16:53 Uhr / Editiert am 30. Juli 2020, 13:29 Uhr / DIE ZEIT Nr. 32/2020, 30. Juli 2020 / 219 Kommentare / 



Als die Schulen wegen der Corona-Pandemie geschlossen wurden, begann ein großes Experiment: Wie gut würde es den Lehrkräften gelingen, ihre Schülerinnen und Schüler aus der Distanz heraus zu unterrichten? Eine Studie, die der ZEIT vorliegt, zeigt nun, dass Deutschland insbesondere beim Mathematik-Unterricht deutlich hinter anderen Ländern zurückbleibt.

Forscher der Universitäten Duisburg-Essen, Utrecht und Antwerpen haben dazu 1706 Mathematik-Lehrkräfte aus Deutschland, den Niederlanden und Flandern (dem niederländischsprachigen Teil Belgiens) befragt. Die Studie ist nicht repräsentativ, aber die Trends sind ernst zu nehmen: 86 Prozent der flämischen und niederländischen Mathe-Lehrer hatten mindestens einmal pro Woche direkten elektronischen Kontakt zu ihren Schülern. Sie haben sie per Videokonferenz, per Live-Chat oder durch die gemeinsame Arbeit an geteilten Dokumenten unterrichtet. In Deutschland waren es nur 47 Prozent. 31 Prozent der deutschen Lehrkräfte haben zudem niemals solche Methoden des sogenannten synchronen Arbeitens genutzt; bei unseren Nachbarn waren das maximal sieben Prozent.



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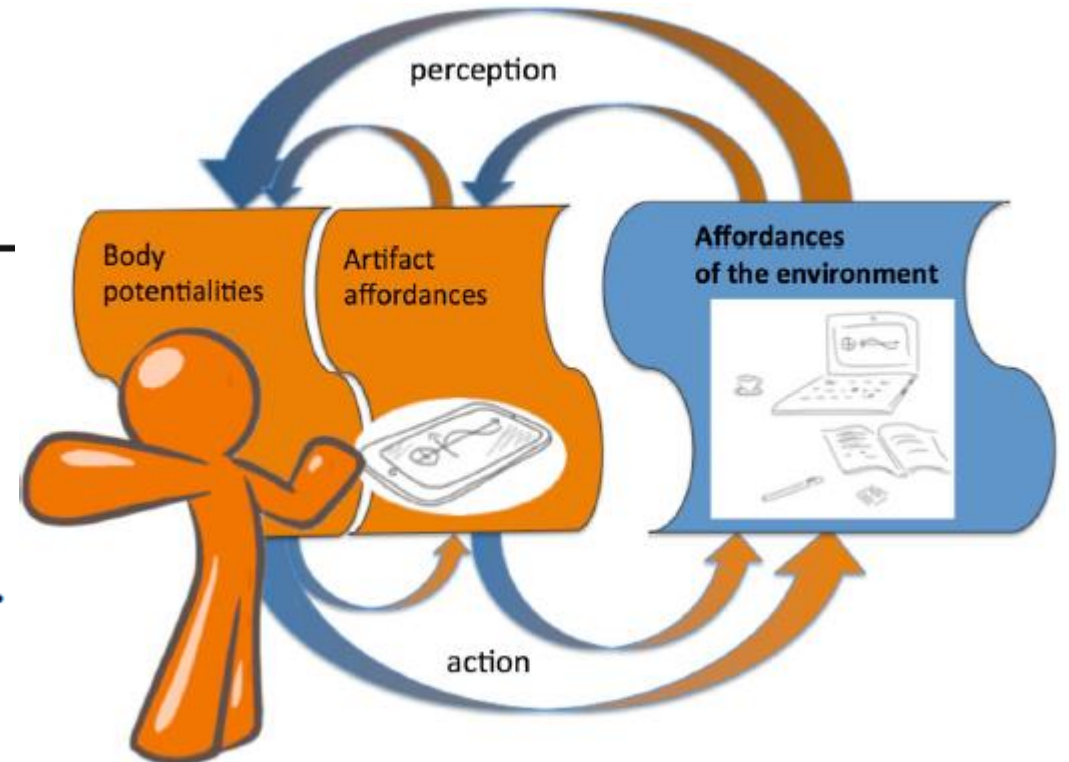
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Present theoretical developments: embodied instrumentation

Educational Studies in Mathematics
<https://doi.org/10.1007/s10649-021-10053-0>

Embodied instrumentation in learning mathematics
as the genesis of a body-artifact functional system

Anna Shvarts¹  • Rosa Alberto¹ • Arthur Bakker¹  • Michiel Doorman¹  •
Paul Drijvers¹ 



(Shvarts et al. 2021)

Teachers
Students

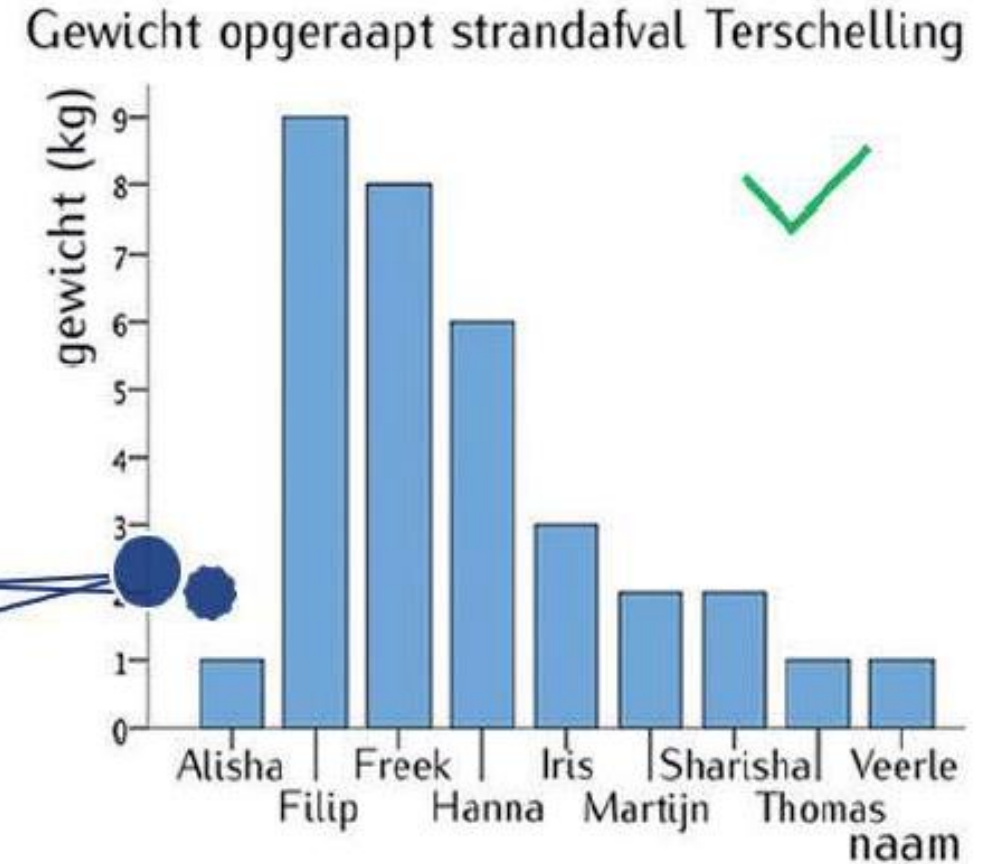
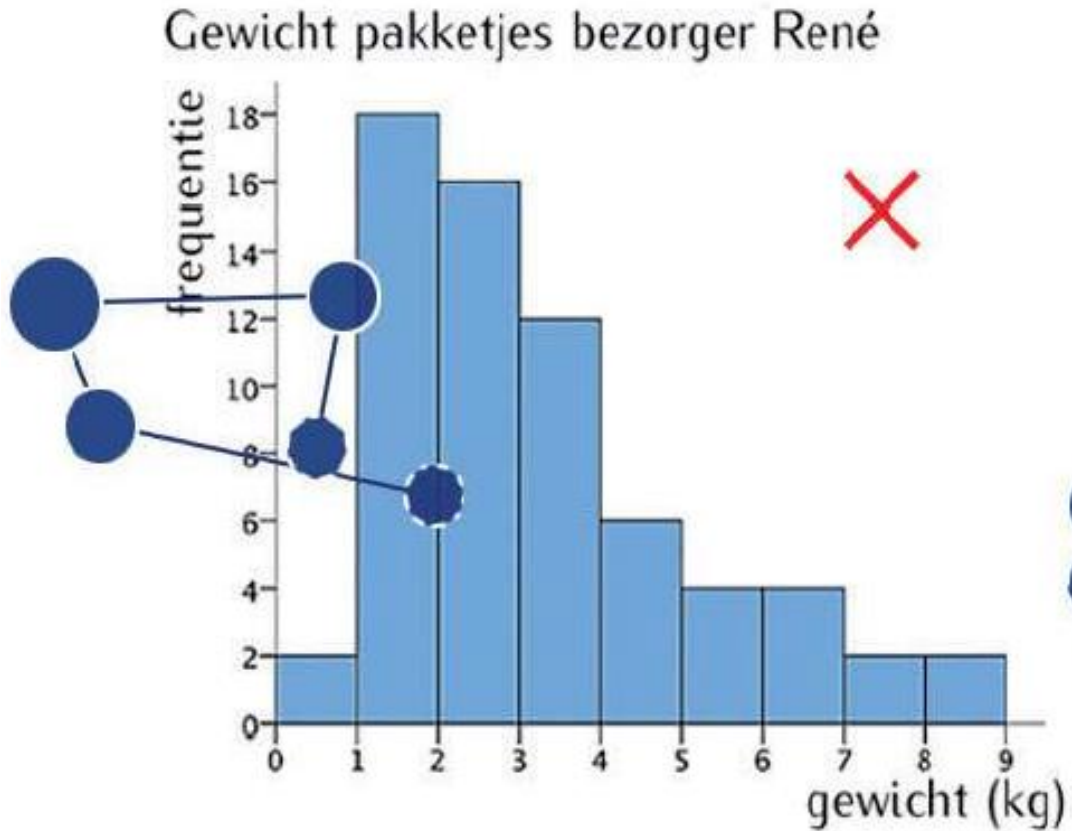
	κ	%A	What went well?				What went badly?				κ	%A	What went well?				What went badly?				
						Total				Total						Total				Total	
(RQ1) Teaching practices																					
- Teaching Activity	.60	87.4	187	385	9	668	14	91	13	118	- Teaching activity	.80	96.8	253	136	23	620	81	16	75	172
- Technology	.59	95.3	40	276	21	337	44	187	8	189	- Technology	.79	98.5	166	33	76	275	38	12	20	65
- Workload	.70	97.7	4	16	0	20	42	76	1	119	- Workload	.87	99.6	9	2	28	39	62	8	50	120
- Creating new digital materials	.84	98.6	62	92	10	164	2	8	0	10	- Materials	.66	99.4	52	5	65	122	10	1	18	29
# Colleagues	.67	99.0	15	10	2	27	1	6	4	11	* Teacher's (digital) skills	.73	99.7	22	5	18	45	4	0	4	8
# Self-management	.32	99.3	5	9	0	14	3	18	0	21	* Comparison F2F - online	.81	99.4	19	18	6	43	13	11	9	33
(RQ2) Didactics																					
- Pedagogy	.57	94.1	32	104	9	145	78	90	19	187	- Pedagogy	.89	99.6	47	7	33	87	38	17	23	78
- Flexibility	.80	99.7	59	17	4	80	8	20	1	29	- Flexibility	.72	99.7	20	8	39	67	2	1	1	4
- Interaction	.59	90.5	37	276	1	325	107	307	42	456	- Interaction student-teacher	.86	98.4	110	36	100	246	137	48	49	234
- Content-specific comments	.90	99.6	3	17	0	20	15	32	1	48	- (no) difficulty with content	.86	99.3	74	4	44	122	107	11	45	163
											* Interaction student-student	.64	99.3	3	2	15	20	17	9	10	36
											* Structure and instruction	.91	99.4	116	30	74	220	28	0	15	43
											* Teacher's aid	.99	99.9	4	2	12	18	38	3	34	75
(RQ3) Assessment																					
- Formative assessment	.55	92.7	32	225	1	270	117	279	36	432	- Formative assessment	.97	99.9	26	7	22	55	9	13	9	31
- Summative assessment	.73	98.8	10	9	3	22	46	79	11	136	- Summative assessment	.99	100	6	2	4	12	24	6	1	31
(RQ4) Situational circumstances																					
- Student equity	.53	95.7	1	26	0	27	16	230	2	248	- Student equity	.92	99.8	3	2	1	6	43	3	28	74
- Technological infrastructure	.60	96.2	5	9	0	14	46	82	2	130	- Technological infrastructure	.82	99.4	10	2	6	18	84	34	78	196
- Well-being	.40	99.7	4	2	0	6	2	12	3	17	- Well-being	.87	99.9	7	2	6	15	10	0	7	17
# Parental work	.68	97.9	1	19	1	21	5	30	0	35											
(RQ5) Student behavior																					
- Student motivation	.60	96.7	21	54	6	81	30	68	8	106	- Motivation and engagement	.91	99.7	6	7	8	21	32	9	18	59
- Student presence	.65	96.7	10	25	3	38	44	79	22	145	- Presence	1	100	0	2	9	11	3	3	15	21
- Student self-management	.63	95.7	52	84	2	138	33	66	10	109	- Self-management	.84	99.0	212	31	115	358	34	14	28	76
											* Disturbance	.99	100	16	17	19	52	9	7	12	28
											* Concentration	1	100	26	8	4	38	48	22	12	82

Color index

What went well	What went badly
Max	Max
Average	Average
Min.	Min.

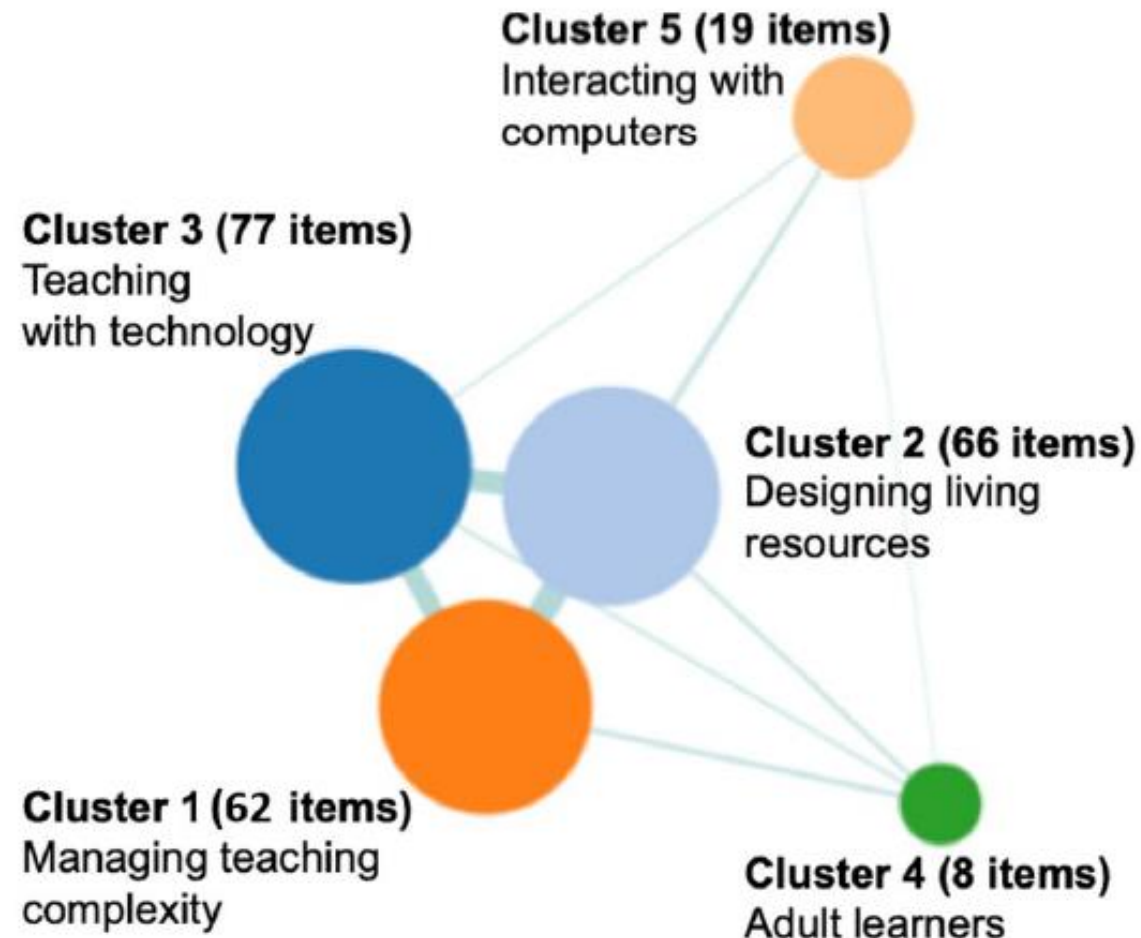
Drivers et al. (under review)

New research methods: eye-tracking



(Boels et al. under review)

New research methods: bibliometric clustering



(Drijvers, Trouche, & Grauwin, 2020)

The present in short

- **Integration of different types of tools**
- **New tools, teaching approaches and research techniques**
- **Limitations / fall back because of ERT / COVID**



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Solve the equation $x^2 - x - 6 = 0$ in different ways.



There are several ways to solve the equation $x^2 - x - 6 = 0$. Here are a few methods:

Method 1: Factoring

One way to solve this equation is to factor it. We need to find two numbers that multiply to -6 and add up to -1 (since the coefficient of x is -1). We can see that -3 and 2 satisfy these conditions. So, we can write:

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

Now, we can use the zero product property and set each factor equal to zero:

$$x - 3 = 0 \text{ or } x + 2 = 0$$

Solving for x in each case, we get:

$$x = 3 \text{ or } x = -2$$

Therefore, the solutions to the equation $x^2 - x - 6 = 0$ are $x = 3$ and $x = -2$.

P

Solve the equation $x^2 - x - 6 = 0$ in different ways.

Method 2: Quadratic Formula

Another way to solve this equation is to use the quadratic formula. The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a , b , and c are the coefficients of the quadratic equation $ax^2 + bx + c = 0$. In this case, $a = 1$, $b = -1$, and $c = -6$. Plugging these values into the quadratic formula, we get:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1 + 24}}{2}$$

$$x = \frac{1 \pm 5}{2}$$

So, the solutions to the equation $x^2 - x - 6 = 0$ are $x = 3$ and $x = -2$, which are the same solutions we got from factoring.

P

Solve the equation $x^2 - x - 6 = 0$ in different ways.

Method 3: Completing the Square

We can also solve this equation by completing the square. To do this, we first move the constant term (-6) to the right side of the equation:

$$x^2 - x = 6$$

Next, we need to add and subtract $(b/2a)^2$ to the left side of the equation, where b is the coefficient of x (-1) and a is the coefficient of x^2 (1):

$$x^2 - x + (-1/2)^2 - (-1/2)^2 = 6$$

Simplifying, we get:

$$(x - 1/2)^2 - 25/4 = 0$$

Now, we can add 25/4 to both sides of the equation:

$$(x - 1/2)^2 = 25/4$$

Taking the square root of both sides, we get:

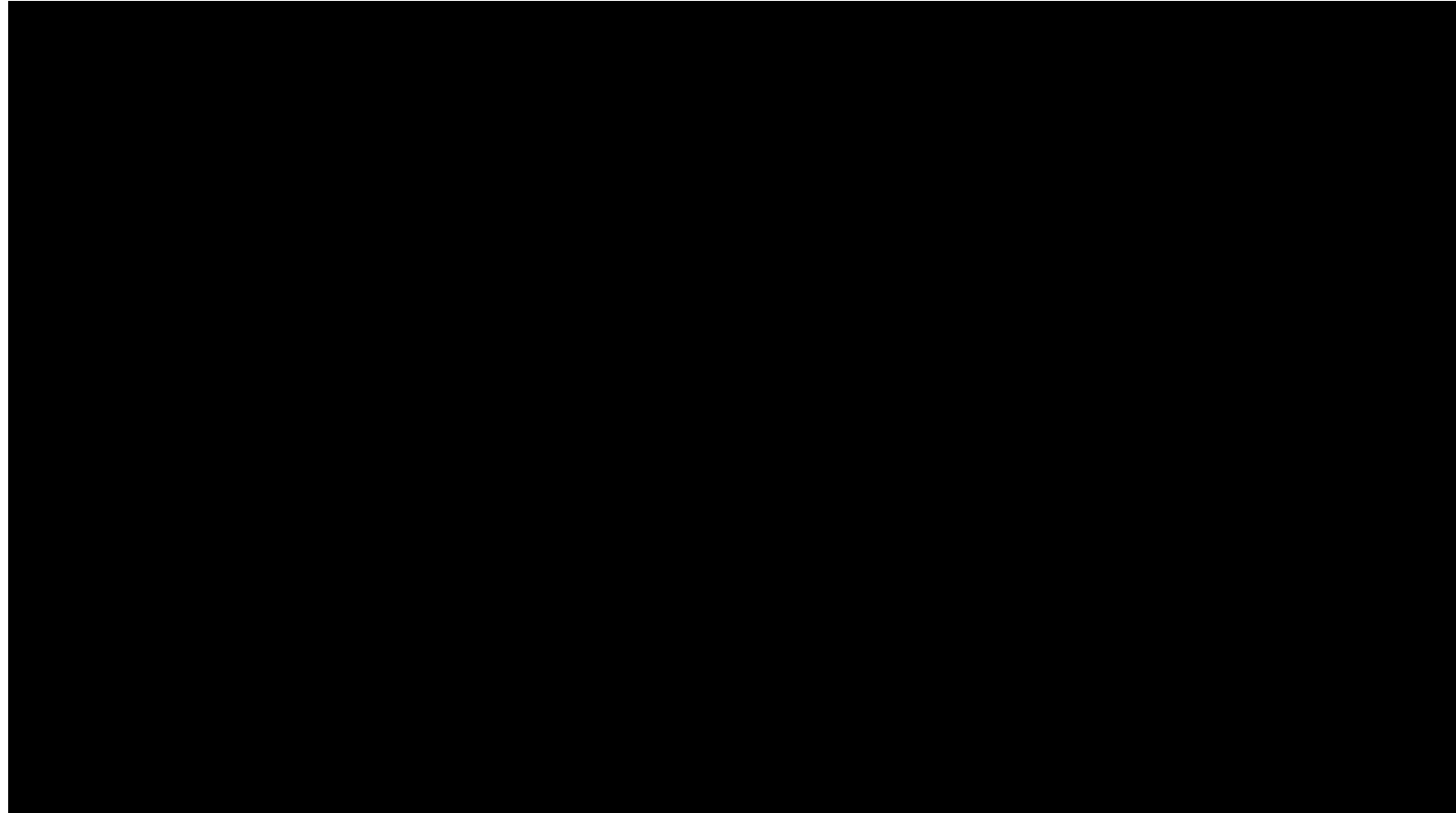
$$x - 1/2 = \pm 5/2$$

Solving for x, we get:

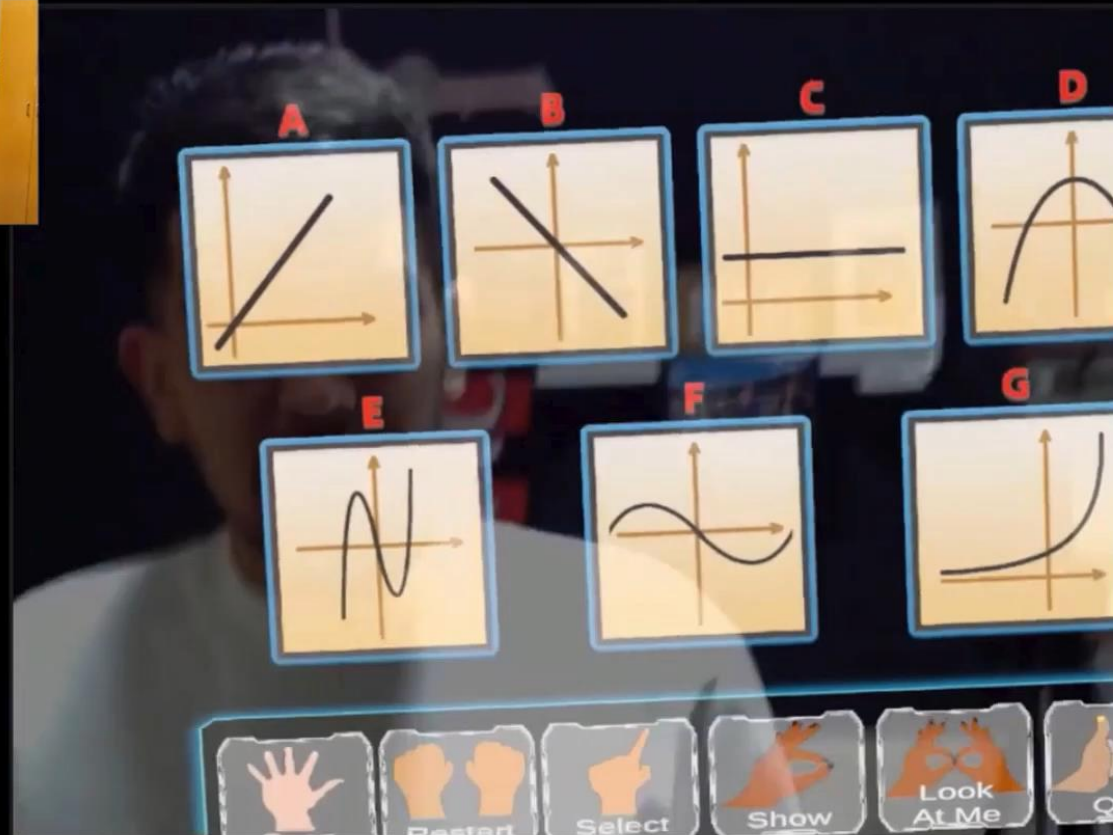
$$x = 1/2 \pm 5/2$$

 Regenerate response

Future technology: Augmented and Virtual Reality (1)



Future technology: Augmented and Virtual Reality (2)



What do you see?



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	Tools	Teaching	Theory
Past			
Present			
Future			<p>THE FUTURE IS HERE: What we know - We are in a world of one-to-one learning paths... ... for better or for learning... ... using technology to enhance or replace traditional instruction.</p> <p>What we should find out: - How does it fit in with our current learning goals? - What are the skills in the digital era? - Can we really do 'competence-based' learning? - What are the ethical principles for learning and teaching in the digital era? - How to make up 'equipment' (equipment research building)?</p>



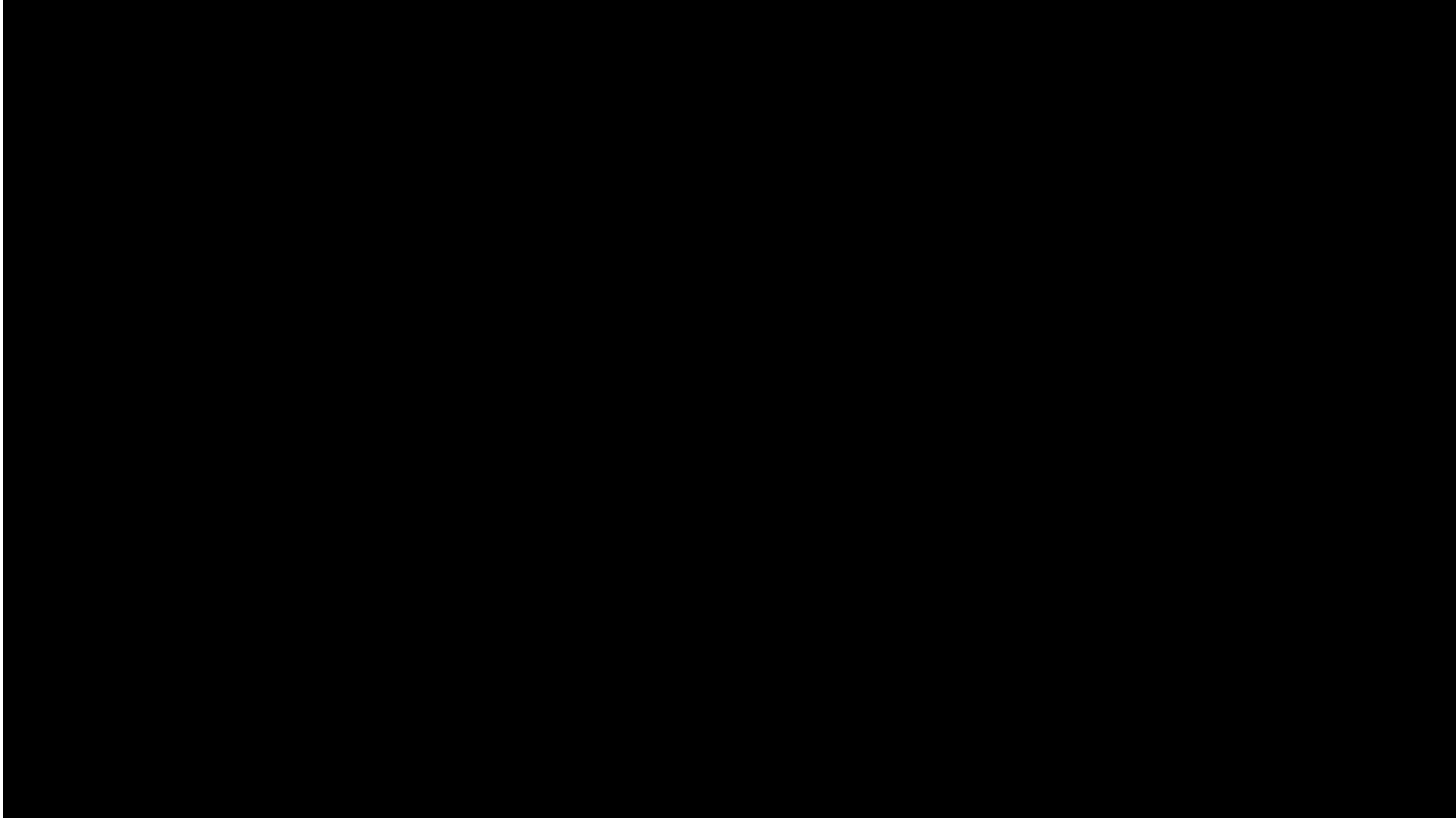
Utrecht University

	Tools	Teaching	Theory
Past			
Present			
Future			<p>THE FUTURE IS NOW</p> <p>What do we know?</p> <ul style="list-style-type: none"> • CEI and its work will continue to learning goals... ... but human core teaching elements... ... using technology to enhance or substitute traditional instruction. <p>What do we think will not?</p> <ul style="list-style-type: none"> • The idea of "flipped" or "inverted" classrooms? • The idea of "blended" or "hybrid" learning? • The idea of "personalized" or "adaptive" learning? • The idea of "micro-credentials" or "badges"? • The idea of "micro-masters" or "micro-degrees"? • The idea of "micro-credentials" or "badges"? • The idea of "micro-masters" or "micro-degrees"? <p>How do we see the future of learning and teaching in the digital era?</p> <p>How do we see the future of learning and teaching in the digital era?</p>

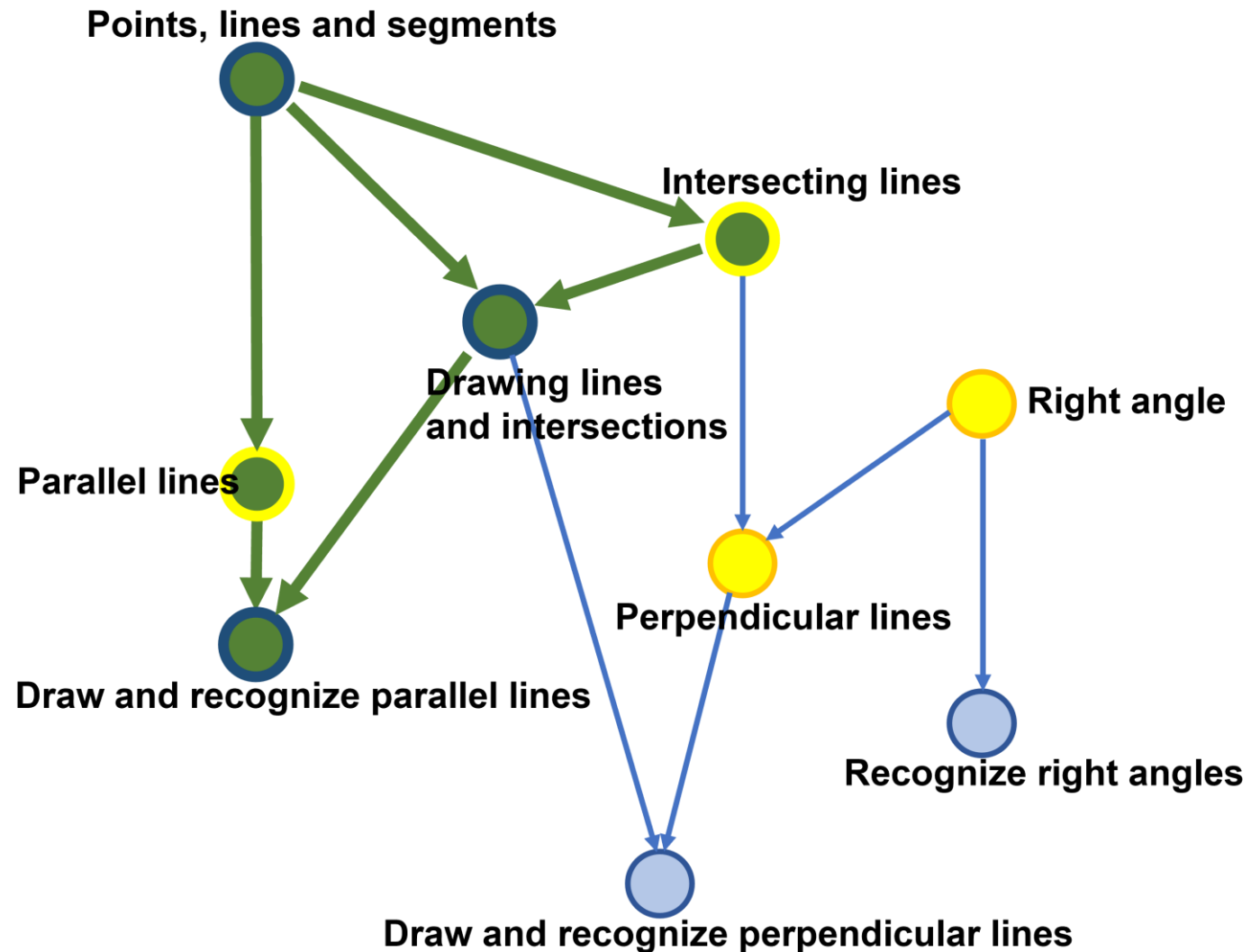
Future teaching: the Virtual Classroom



Future teaching: embodied design



Future teaching: formative assessment





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	Tools	Teaching	Theory
Past			
Present			
Future			<p>THE FUTURE IS HERE!</p> <p>What do we know?</p> <ul style="list-style-type: none"> • CEI will be used in our field in learning goals... ... but human care teaching remains... ... using technology to enhance or substitute student-led instruction. <p>What do we think will not?</p> <ul style="list-style-type: none"> • The idea of CEI (focus on each individual learning path?) • What are the skills in the digital era? • Can we really be "competence-based" at all? • What are theoretical principles for learning and teaching in the digital era? • How to redesign "equipment" (classroom research building)?



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	Tools	Teaching	Theory
Past			
Present			
Future			<p>THE FUTURE IS HERE!</p> <p>What do we know?</p> <ul style="list-style-type: none"> • VR can be used to contribute to learning goals... ... but human core teaching elements... ... still require measurement for validation of student learning outcomes. <p>What do we think will not?</p> <ul style="list-style-type: none"> • The idea of VR replacing or even supplementing learning goals? • What are the core skills in the digital era? • Can we really do "competence-based learning" at all? • What are the ethical principles for learning and teaching in the digital era? • How to make up "equipment" (classroom research building)?

THE FUTURE IN SHORT:

What we know:

- DT use in math ed can lead to learning gains...
- ... but invites new teaching formats...
- ... taking into account the subtleties of student-tool interaction.

What we should find out:

- How does DT impact on math curricula and learning goals?
- What are (basic) skills in the digital era?
- Can we really do "concepts before/above skills"?
- What new theoretical paradigms for learning and teaching in the digital era?
- How to scale up / implement / disseminate research findings?



Utrecht University



CERME 13

13TH CONGRESS OF THE EUROPEAN SOCIETY
FOR RESEARCH IN MATHEMATICS EDUCATION

10-14 July 2023
Budapest
Hungary

**Zooming in and zooming out:
the color chart of digital
technology in mathematics
education**
Paul Drijvers

Thank you for your attention!

Copenhagen, 2023-03-17

www.matematikdidaktik.dk